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MATH43021 - 2011/2012

General Information

- Title: Set Theory
- Unit code: MATH43021
- Credits: 15
- Prerequisites: MATH33001 (Predicate Logic) useful but not essential.
- Co-requisite units: None.
- School responsible: Mathematics
- Members of staff responsible: Dr. A. Vencovska

Specification

Aims

To introduce students to the elements of set theory and its role as a foundation for mathematics.

Brief Description of the unit

The notion of set is the fundamental notion of modern mathematics. In the late 19th century Georg Cantor, who is now seen as the father of modern set theory, developed a revolutionary concept of transfinite numbers that can be used to compare the 'sizes' of possibly infinite sets. A naive approach to set theory leads to paradox and it was left to Zermelo to propose an axiomatic approach that puts set theory on a sound rigorous basis. Axiomatic set theory can be viewed as a foundation of mathematics in the following sense: all mathematical notions can be defined in purely set theoretical terms and their properties can be proved using only the set theoretical axioms. Furthermore the language of set theory has played a central unifying role in modern mathematics.

The course unit will present Cantor's theory of transfinite numbers, the axiom system ZF (Zermelo-Fraenkel set theory), the Axiom of Choice and the Continuum Hypothesis.

Learning Outcomes

On successful completion of the course unit students will be able to demonstrate facility with the notions of elementary set theory and show an understanding of how axiomatic set theory can be a foundation for mathematics.

Future topics requiring this course unit

None.

Syllabus

1. Naïve set theory, review of basic set theoretic notation and operations. Finite and countable sets, cardinal numbers and their ordering , the arithmetic of cardinal numbers, comparison of cardinal numbers, Schroeder-Bernstein theorem. (9 lectures)
2. Russell's Paradox, Zermelo-Fraenkel set theory ZF, ordered pairs, Cartesian product, definition of a function. (4 lectures)
3. Well-orderings, ordinals, transfinite induction. (7 lectures)
4. Set theoretical definition of natural, rational and real numbers. (2 lectures)
5. Axiom of Choice, Zorn's lemma, the Well-Ordering theorem. The Spheres Paradox. The Continuum Hypothesis. (11 lectures)

Textbooks

There is no recommended textbook to cover the course. The following are some good books to consult.

- A. G. Hamilton, *Numbers, Sets and Axioms*, CUP Press (in paperback).
- Y. N. Moschovakis, *Notes on Set Theory*, Springer-Verlag Undergraduate Texts in Mathematics.

- H. B. Enderton, *Elements of Set Theory*, Academic Press.

Teaching and learning methods

Three lectures a week plus one weekly office hour. In addition students should expect to do at least seven hours private study each week for this course unit.

Assessment

Coursework: two take home tests; weighting 10% each,
End of semester examination: two and a half hours; weighting 80%

Arrangements