MATH46052 - 2012/2013

General Information

- Title: Approximation Theory and Finite Element Analysis
- Unit code: MATH46052
- Credits: 15
- Prerequisites: MATH20401 PDEs and vector calculus. MATH20101 Real and Complex Analysis
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. David Silvester

Specification

Aims

To give an understanding of the fundamental methods and theoretical basis of approximation. To provide students with the technical tools enabling them to solve practical elliptic PDE problems using the finite element method.

Brief Description of the unit

This course unit covers the theory of approximation and applications to the numerical solution of linear elliptic partial differential equations (PDEs) using finite element approximation methods. Such methods are universally used to solve practical problems associated with physical phenomena in complex geometries. The emphasis is on assessing the accuracy of the approximation using a priori and a posteriori error estimation techniques. Practical issues will be illustrated with MATLAB using the IFISS software toolbox.

Learning Outcomes

On successful completion of this course unit students will

- understand notions of best approximation in different norms and be able to find the best approximation;
- understand the concepts of weak and classical solutions of elliptic boundary value problems;
- understand the concept of piecewise polynomial approximation in two dimensions, and have an appreciation for the underlying error analysis;
- have an appreciation of the computational issues that arise when solving convection-diffusion problems;

Future topics requiring this course unit

None

Syllabus

1. **Basics.** Review of basic functional analysis concepts: norms, inner-products. Sobolev spaces. Weak derivatives. Lax-Milgram lemma. [3]
2. **Linear approximation.** Best approximation in L_p norms. Existence and uniqueness. Choice of norm in practical curve fitting. Least squares approximation and normal equations. Orthogonal basis functions. Overview of the cases $p=1$ and $p=\infty$. Choice of linear approximating functions. Polynomials, orthogonal polynomials, Chebyshev polynomials, Spline functions. Surface fitting by polynomials and splines, including the thin plate spline and radial basis functions.[8]
3. **Finite element methods for the diffusion equation.** Affine mappings. Linear, bilinear, quadratic and biquadratic approximation. Finite element assembly process. Properties of the discrete equation system. A priori error bounds: best approximation in energy, H^1 error bounds. H^2 regularity and singular problems. A posteriori error bounds. Local error estimators. Self adaptive refinement strategies. [14]
4. **Finite element methods for the convection-diffusion equation.** Well-posedness. Weak formulation. Galerkin approximation. The streamline-diffusion method. A priori and a posteriori error bounds. Self-adaptive refinement strategies for resolving layers. [5]

Textbooks

- Michael J. D. Powell, *Approximation Theory and Methods*, ISBN 978-0-521-295149 (pbk) Cambridge University Press, Cambridge, 1981.
- Howard Elman, David Silvester and Andy Wathen, *Finite Elements and Fast Iterative Solvers*, ISBN 0-19-852868-X (pbk) Oxford University Press, Oxford 2005
- Dietrich Braess, *Finite Elements: Theory, Fast Solvers and Applications in Solid Mechanics*, ISBN 978-0-521-70518-9 (pbk) Cambridge University Press, Cambridge, third edition, 2007.

Teaching and learning methods

30 lectures (two or three lectures per week) with a weekly examples class. In addition should expect to spend at least six hours each week on private study.

Assessment

Mid-semester coursework: 25%
End of semester examination: three hours weighting 75%

Arrangements