MANCHESTER 1824

MATH32012 - 2012/2013

General Information

- Title: Commutative Algebra
- Unit code: MATH32012
- Credits: 10
- Prerequisites: MATH20201 *Algebraic Structures 1,* MATH20212 *Algebraic Structures 2.*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr Yuri Bazlov

Specification

Aims

The course unit will deepen and extend students' knowledge and understanding of commutative algebra. By the end of the course unit the student will have learned more about familiar mathematical objects such as polynomials and algebraic numbers, will have acquired various computational and algebraic skills and will have seen how the introduction of structural ideas leads to the solution of mathematical problems.

Brief Description of the unit

The central theme of this course is factorisation (theory and practice) in commutative rings; rings of polynomials are our main examples but there are others, such as rings of algebraic integers.

Polynomials are familiar objects which play a part in virtually every branch of mathematics. Historically, the study of solutions of polynomial equations (algebraic geometry and number theory) and the study of symmetries of polynomials (invariant theory) were a major source of inspiration for the vast expansion of algebra in the 19th and 20th centuries.

In this course the algebra of polynomials in *n* variables over a field of coefficients is the basic object of study. The course covers fairly recent advances which have important applications to computer algebra and computational algebraic geometry (Gröbner bases - an extension of the Euclidean division algorithm to polynomials in 2 or more variables), together with a selection of more classical material.

Learning Outcomes

On successful completion of this course unit students will be able to demonstrate

- facility in dealing with polynomials (in one and more variables);
- understanding of some basic ideal structure of polynomial rings;
- appreciation of the subtleties of factorisation into prime and irreducible elements;
- ability to compute generating sets and Gröbner bases for ideals in polynomial rings;
- ability to relate polynomials to other algebraic structures such as algebraic varieties;
- ability to solve problems relating to the factorisation of polynomials, irreducible polynomials and polynomial equations in several variables.

Future topics requiring this course unit

None, though the material connects usefully with algebraic geometry and Galois theory.

Syllabus

1. Ideals in commutative rings: euclidean rings, principal ideal rings, noetherian rings. [4]
2. Ideals in polynomial rings: monomial orderings, Gröbner bases, Hilbert's basis theorem. [4]
3. Computing ideals in polynomial rings: division algorithm, Buchberger's algorithm. [4]
4. Factorisation: irreducible and prime elements, unique factorisation domains, Gauss's Lemma, Eisenstein's criterion, fields of fractions. [6]

5. Zero sets of polynomials: algebraically closed fields, affine varieties, radical of an ideal, elimination method, the Nullstellensatz. [4]

Textbooks

[1,2] are are useful general references on algebra though neither covers more advanced content of the course such as Gröbner bases. For Gröbner bases see [3]. A treatment of factorisation as well as Gröbner bases, with many exercises, is given in [4]. [5] is a new textbook which combines Gröbner bases with more traditional material and contains some practical examples.

1. R.B.J.T. Allenby, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, Edward Arnold, 0-3405-4440-6.
2. J.B. Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley,1994, 0-201-59291-6.
3. D.A. Cox, J. Little and D. O'Shea, *Ideals, Varieties and Algorithms* (3rd edition), Springer 2007, 978-0387356501.
4. D.A. Dummit, R.M. Foote, *Abstract Algebra* (3rd edition), Wiley & Sons 2003, 978-0471433347. [Chapters 7, 8, 9 and 15.]
5. N. Lauritzen, *Concrete Abstract Algebra*, Cambridge University Press 2003, 978-0521534109. [Chapters 3, 4 and 5.]

Teaching and learning methods

Two lectures and one examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

Course notes will be provided, as well as examples sheets and solutions. The notes will be concise and will need to be supplemented by your own notes taken in lectures, particularly of worked examples.

Assessment

Coursework: weighting 20%
End of semester examination: two hours weighting 80%

Arrangements