MATH41051 - 2012/2013

General Information

- Title: Introduction to Topology
- Unit code: MATH41051
- Credits: 15
- Prerequisites: MATH20122 *Metric Spaces* is desirable but not essential
- Co-requisite units: None
- School responsible: Mathematics
- Member of staff responsible: Prof. Peter Eccles

Specification

Aims

This lecture course aims to introduce students to the basic concepts of topological spaces and continuous functions, and to illustrate the techniques of algebraic topology by means of the fundamental group.

Brief Description of the unit

This course unit is concerned with the study of topological spaces and their structure-preserving functions (continuous functions). Topological methods underpin a great deal of present day mathematics and theoretical physics. Topological spaces are sets which have sufficient structure so that the notion of continuity may be defined for functions between topological spaces. This structure is not defined in terms of a distance function but in terms of certain subsets known as open subsets which are required to satisfy certain basic properties. Continuous functions may stretch or bend a space and so two spaces are considered to be topologically equivalent if one can be obtained from the other by by stretching and bending: for this reason topology is sometimes called rubber sheet geometry.
The first half of the course unit introduces the basic definitions and standard examples of topological spaces as well as various types of topological spaces with good properties: pathconnected spaces, compact spaces and Hausdorff spaces . The second half introduces the fundamental group and gives some standard applications of the fundamental group of a circle.
MATH41051 is enhanced by additional reading on topics closely related to topics covered in the lectures.
It should be noted that this syllabus has been revised for 2012-2013 and the second half covers topics previously included in MATH31072.

Learning Outcomes

On successful completion of this course unit students will be able to

- prove that certain subsets of Euclidean space are homeomorphic (topologically equivalent) by constructing a homeomorphism;
- understand the notions of path-connectedness and path-component and be familiar with some standard applications;
- determine whether a collection of subsets of a set determines a topology;
- understand the notions of continuity at a point for maps of topological spaces and the notions of interior and closure of a subset of a topological space;
- use the definitions of the subspace topology, the product topology and the quotient topology, understand the use and proof of their universal properties, and be familiar with standard examples such as topological surfaces;
- recognize whether or not a subset of a topological space is compact and be familiar with the basic properties of compact subsets and their proofs;
- recognize whether or not a topological space is Hausdorff and be familiar with the basic properties of Hausdorff spaces and their proofs;
- understand the notion of a group action on a topological space, and be familiar with the topological properties of the quotient space of a finite group action on a compact Hausdorff space;

- understand the definition of the fundamental group and its functorial properties; calculate the fundamental group of the circle using its univeral covering space and have seen some standard applications;
- understand the notion of a covering space and the relationship with the fundamental group.

Future topics requiring this course unit

MATH31062/41062 *Differentiable Manifolds*.

Syllabus

1. **Topological equivalence**: the topological equivalence of subsets of Euclidean spaces, path-connected sets and distinguishing subsets of Euclidean spaces using the cut point principle. Standard applications of path-connectedness such as the Pancake Theorem. [3 lectures]
2. **Topological spaces**: definition of a topology on a set, a topological space and a continuous function between topological spaces; closed subsets of a topological space; a basis for a topology. [2]
3. **Topological constructions**: subspaces, product spaces, quotient spaces; definitions and basic properties; standard examples including the cylinder, the torus, the Möbius band, the projective plane and the Klein bottle. [5]
4. **Compactness**: open coverings and subcoverings, definition of a compact subset of a topological space; basic properties of compact subsets; compact subsets in Euclidean spaces (the Heine-Borel Theorem). [2]
5. **Hausdorff spaces**: definition and basic properties of Hausdorff spaces; a continuous bijection from a compact space to a Hausdorff space is a homeomorphism, quotient spaces of compact Hausdorff spaces. [2]
6. **The fundamental group**: equivalent paths, the algebra of paths, definition of the fundamental group and dependence on the base point. [3]
7. **The fundamental group of the circle**: the path lifting theorem for the standard cover of the circle, the degree of a loop in the circle, the fundamental group of the circle, standard applications: the Brouwer Non-Retraction Theorem, the Brouwer Fixed Point Theorem, the Fundamental Theorem of Algebra, the Hairy Ball Theorem. [5]

Additional reading for MATH41051

A. **Neighbourhoods**, interior and closure: interior points and neighbourhoods, using neighbourhoods to define a topology, continuity at a point, closure points.

B. **Group actions on spaces**: definition of a continuous group action on a topological space; the quotient space of a group action; a quotient space of a finite group action on a compact Hausdorff space is compact Hausdorff.

C. **Covering spaces**: definition of a covering space; path-lifting for covering spaces; group actions and covering spaces; examples of using covering spaces to calculate a fundamental group: the fundamental groups of the torus, the projective plane and the Klein bottle; the Borsuk-Ulam Theorem.

Textbooks

The first three of the following books contains most of the material in the course with the third a little more advanced than the first two. The fourth book contains most of material in the first half of the course and relates topological spaces to metric spaces.

- M. A. Armstrong. Basic Topology, Springer 1997.
- C. Kosniowski, A First Course in Algebraic Topology, Cambridge University Press 1980.
- J. R. Munkres, Topology, Prentice-Hall 2000 (second edition).
- W. A. Sutherland, Introduction to Metric and Topological Spaces, Oxford University Press 2009 (second edition).

Teaching and learning methods

Three classes each week which will include opportunities to discuss problems from the Problems Sheets. Additional tutorials to discuss the additional reading will be organized as required. In addition students should expect to spend at least seven hours each week on private study for this course unit.

Assessment

Mid-semester coursework: weighting 10%

End of semester examination: three hours weighting 90%

Arrangements