



## MATH37001 - 2012/2013

### General Information

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- Title: Martingales with Applications to Finance
- Unit code: MATH37001
- Credits: 10
- Prerequisites: MATH20701
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. [T. Zhang](#)

## Specification

### Aims

To provide a firm grasp of a range of basic concepts and fundamental results in the theory of martingales and to give some simple applications in the rapidly developing area of financial mathematics.

### Brief Description of the unit

An introduction to a circle of ideas and fundamental results of the theory of martingales, which play a vital role in stochastic calculus and in the modern theory of finance.

### Learning Outcomes

On successful completion of this course unit students will

- have a good understanding of the basic concept of integration with respect to a probability measure and the basic properties of fair games;
- be able to answer simple questions on martingales;
- experience applications of stochastic processes in discrete financial models.

### Future topics requiring this course unit

The material of this course will be used in the 3rd and 4th year courses on stochastic processes and stochastic calculus.

### Syllabus

1. Probability spaces, events,  $\sigma$ -fields, probability measures and random variables. Integration with respect to a probability measure. Convergence theorems (dominated, monotone and Fatou). [5]
2. Conditional expectations. Fair games and martingales, submartingales and supermartingales. Doob decomposition theorem. Stopping times and the optional sampling theorem. The upcrossing inequality and the martingale convergence theorem. The Doob maximal inequality and the martingale modification theorem. [13]
3. Applications. Discrete time random models in financial markets. Price processes, self-financing portfolio and value processes. Arbitrage opportunities and equivalent martingale measures. Completeness of the markets. Options and option pricing. [6]

### Textbooks

- O. Kallenberg, *Foundations of Modern Probability*, Springer-Verlag, 2001.
- N. H. Bingham and R. Kiesel, *Risk-Neutral Valuation*, Springer-Verlag, 1998.
- D. Williams, *Probability with Martingales*, Cambridge Univ. Press, 1991.

- A. N. Shiryaev, *Probability*, Springer-Verlag, 1996.

## **Teaching and learning methods**

Two lectures and one examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

## **Assessment**

End of semester examination: two hours weighting 100%

## **Arrangements**