MATH36001 - 2012/2013

General Information

- Title: Matrix Analysis
- Unit code: MATH36001
- Credits: 10
- Prerequisites: MATH10202, MATH10212 *Linear Algebra*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr Stefan Guettel

Specification

Aims

To introduce students to matrix analysis through the development of essential tools such as the Jordan canonical form, Perron-Frobenius theory, the singular value decomposition, and matrix functions.

Brief Description of the unit

This course unit is an introduction to matrix analysis, covering both classical and more recent results that are useful in applying matrix algebra to practical problems. In particular it treats eigenvalues and singular values, matrix factorizations, function of matrices, and structured matrices. It builds on the first year linear algebra course. Apart from being used in many areas of mathematics, Matrix Analysis has broad applications in fields such as engineering, physics, statistics, econometrics and in modern application areas such as data mining and pattern recognition. Examples from some of these areas will be used to illustrate and motivate some of the theorems developed in the course.

Learning Outcomes

On successful completion of this course unit students will

- be familiar enough with matrix analysis and linear algebra that they can effectively use the tools and ideas of these fundamental subjects in a variety of applications,
- understand the importance of spectral decomposition, Schur decomposition, Jordan canonical form and singular value decomposition,
- understand the role of matrix functions in solving differential and algebraic equations,
- understand how to exploit the structure in certain classes of matrices.

Future topics requiring this course unit

MATH46101 *Numerical Linear Algebra*

Syllabus

1. **Basics:** Summary/recap of basic concepts from linear algebra including matrices and vectors, determinants, singularity of matrices, rank. [2 lectures]
2. **Theory of eigensystems:** Eigenvalues, eigenvectors, and invariant subspaces; reduction of square matrices to simpler form including the Schur decomposition, spectral decomposition for normal matrices and the Jordan canonical decomposition; minimal and characteristic polynomials, Cayley-Hamilton Theorem; Sylvester's inertia theorem. [6]
3. **Norms:** Vector norms and matrix norms, bounds for eigenvalues, Gershgorin theorem. [2]
4. **Singular value decomposition (SVD):** Projectors; pseudo-inverses; application to linear least squares; polar decomposition. [3]
5. **Nonnegative matrices and related results:** Irreducible matrices; Perron-Frobenius theorem; diagonally dominant matrices. [2]
6. **Matrix functions:** Definitions; the matrix exponential function and application to the solution of differential equations and higher order equations; difference equations and matrix powers. [5]

7. **Kronecker product.** Definition, properties and application to the solution of Sylvester's equation (if time). [2]

Textbooks

- Roger A. Horn and Charles R. Johnson. *Matrix Analysis*. Cambridge University Press, 1985.
- Peter Lancaster and Miron Tismenetsky. *The Theory of Matrices*. Academic Press, London, second edition, 1985.
- Alan J. Laub. *Matrix Analysis for Scientists and Engineers*. SIAM, Philadelphia, PA, 2005.
- Carl D. Meyer. *Matrix Analysis and Applied Linear Algebra*. SIAM, Philadelphia, PA, 2000.
- James M. Ortega. *Matrix Theory: A Second Course*. Plenum Press, New York, 1987.

Teaching and learning methods

Two lectures per week, with a weekly examples class. In addition students should expect to do at least four hours private study each week for this course unit.

Assessment

Mid-semester test: weighting 20%
Two hours end of semester examination: weighting 80%

Arrangements

- Online course materials are available for this unit.