MATH20122 - 2012/2013

General Information

- Title: Metric Spaces
- Unit code: MATH20122
- Credit rating:10
- Level: 2
- Pre-requisite units: MATH20101 or MATH20111
- Co-requisite units:
- School responsible: Mathematics
- Members of staff responsible: Prof. N. Ray

Unit specification

Aims

The programme unit aims to introduce the basic ideas of metric spaces.

Brief description

A metric space is a set together with a good definition of the distance between each pair of points in the set. Metric spaces occur naturally in many parts of mathematics, including geometry, fractal geometry, topology, functional analysis and number theory. This lecture course will present the basic ideas of the theory, and illustrate them with a wealth of examples and applications.

This course unit is strongly recommended to all students who intend to study pure mathematics and is relevant to all course units involving advanced calculus or topology.

Intended learning outcomes

On completion of this unit successful students will be able to:

- deal with various examples of metric spaces;
- have some familiarity with continuous maps;
- work with compact sets in Euclidean space;
- work with completeness;
- apply the ideas of metric spaces to other areas of mathematics.

Future topics requiring this course unit

A wide range of course units in analysis, dynamical systems, geometry, number theory and topology.

Syllabus

1. **Basic Definitions.** Euclidean metric, taxicab metric, discrete metric, edge metric, word metric, sup metric, L^1 metric, Hausdorff metric, l^2 metric, product metrics. Examples. [4 lectures]
2. **Open and Closed Sets.** Interior, closure, sequences and convergence, frontier. Denseness. Equivalent metrics. Examples. [4]
3. **Uniform Convergence.** Sequences of continuous functions. Examples. [2]
4. **Continuous maps.** Extending the elementary definition. Relationship with open sets, sequences. Examples [4]
5. **Compactness.** Open coverings. Continuous maps on compact sets. Compactness in Euclidean space. [4]
6. **Completeness.** Cauchy sequences. The Contraction Mapping Theorem, Examples. [3]

Textbooks

Two books are particularly relevant. The first is

- Wilson A. Sutherland, *Introduction to Metric and Topological Spaces*, Oxford University Press (Second Edition) 2009

which contains almost all the material in the course, is beautifully written, and is highly recommended. Copies are available to purchase in Blackwells, and to borrow from the JRUL. For an alternative view, try

- Micheal O'Searcoid, *Metric Spaces*, Springer 2006.

Learning and teaching processes

Two lectures and one Feedback Examples Class each week. In addition, students should expect to spend at least four hours each week on private study for this course unit.

Assessment

Coursework; Weighting within unit 20%
2 hours end of semester examination; Weighting within unit 80%

Arrangements