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MATH31082/MATH41082 - 2012/2013

General Information

- Title: Riemannian Geometry
- Unit code: MATH31082/MATH41082
- Credit rating: 10 (MATH31082), 15 (MATH41082),
- Level: 3
- Pre-requisite units: MATH20222 *Introduction to Geometry,* MATH20132 *Calculus of Several Variables* MATH31062 *Differentiable Manifolds (optional)*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. H. Khudaverdyan

Unit specification

Aims

The programme unit aims to introduce the basic ideas of Riemannian geometry.

Brief description

Basis ideas of Riemannian geometry such as Riemannian metric, covariant differentiation, geodesics and curvature belong to the core of mathematical knowledge and are widely used in applications that range from general relativity in physics to mechanics and engineering. Besides that, this subject is one of the most beautiful in mathematics, containing such gems as Gauss's Theorema Egregium and the Gauss—Bonnet Theorem providing a link with the topology of surfaces.The course introduces these ideas, building on the course unit MATH20222 Introduction to Geometryand is complementary to the course unit MATH31061 *Differentiable Manifolds* (it is not required but is beneficial to take this course).

Intended learning outcomes

On completion of this unit successful students will be able to:

· deal with various examples of Riemannian metrics;
· work practically with connection and curvature;
· appreciate the relation between geodesics and variational principle;
· apply the ideas of Riemannian geometry to other areas.

Future topics requiring this course unit

Riemannian geometry is used in almost all areas of mathematics and its applications, including physics and engineering.

Syllabus

• ***RIEMANNIAN METRIC ON A DOMAIN OF R^n.***

The notion of Riemannian metric in a domain of $\boldsymbol{R}^n$.
Angle and length of tangent vetors. Arclength of a curve.
Examples of metric: sphere and other quadrics in $\boldsymbol{R}^3$; Lobachevsky plane.
Expression of metric in different coordinates.
Volume element corresponding to Riemannian metric.

• ***COVARIANT DIFFERENTIATION.***

Definition of a covariant derivative. Expression in local coordinates.
Christoffel symbols.
Examples: covariant differentiation in $\boldsymbol{R}^n$ in curvilinear coordinates.
Covariant differentiation on surfaces in $\boldsymbol{R}^3$.
Relations between covariant differentiation and Riemannian metric. Levi-Civita connection.

• ***GEODISICS AND PARALLEL TRANSPORT.***

Idea of parallel transport. Infinitesimal parallel transport.
Equation of parallel transport. Geodesics.
Geodesics and Riemannian metric. Variational principle for geodesics.
Examples of geodesics.

• ***THEORY OF SURFACES.***

Induced Riemannian metric (First quadratic form).
Gauss-Weingarten derivation formulae.
Second quadratic form and Shape (Weingarten) operator.
Theorema Egregium.

• ***CURVATURE TENSOR.***

Infinitesimal parallel transport over a closed contour.
Definition of curvature tensor.
Gaussian curvature of surfaces and scalar curvature.
Application in Gravity theory. Einstein –Gilbert equations.

•***GAUSS—BONNET THEOREM.***

Triangulation of surfaces and Euler characteristic. Examples.
Gauss-Bonnet Theorem.
Two-dimensional gravity.

Textbooks

No particular textbook is followed. Students are advised to keep their own lecture notes. There are many good sources available treating various aspects of differentiable manifolds on various levels and from different viewpoints. Below is a list of texts that may be useful. More can be found by searching library shelves.

- R. Abraham, J. E. Marsden, T. Ratiu. Manifolds, tensor analysis, and applications. Springer-Verlag, 1996. **ISBN** 0387967907.
- B.A. Dubrovin, A.T. Fomenko, S.P. Novikov. Modern geometry, methods and applications. Part I: The Geometry of Surfaces, Transformation Groups, and Fields, Vol. 93, 1992,

- Barret O’ Neill. Elementary Differential geometry, Revised Second Edition, Academic Press (Elsevier), 2006, **ISBN-10:** 0120887355.

Learning and teaching processes

Three classes each week which will include opportunities to discuss problems from the Problems Sheets. In addition students should expect to spend at least four hours each week on private study for this course unit (seven hours for MATH41081 + additional handouts).

Assessment

Coursework (take-home) 20%
End of Semester exam, 2 hours duration (3 hours for MATH41082), 80%.

Arrangements