MANCHESTER 1824

MATH45051 - 2012/2013

General Information

- Title: Singularities, Bifurcations and Catastrophes
- Unit code: MATH45051
- Credit rating: 15
- Level: 4
- Pre-requisite units: MATH20201 Algebraic Structures I, MATH20132 Calculus of several variables.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. J. Montaldi

Unit specification

Aims

This lecture course aims to introduce students to the fundamental ideas of classifying and understanding singularities of maps and their unfoldings, and to enable them to apply these notions to bifurcation problems.

Brief description

A large part of Singularity Theory concerns the local structure of maps $\boldsymbol{R}^n$ to $\boldsymbol{R}^p$ or $\boldsymbol{C}^n$ to $\boldsymbol{C}^p$, particularly when the conditions of the inverse or implicit function theorems fail, and how this structure changes as the map is deformed. According to which aspects of the map are considered important, there are different equivalence relations on the space of all such maps that can be used to provide classifications. Singularity theory has many applications, and in particular to the study of bifurcations, and is also of interest in its own right.

Intended learning outcomes

On successful completion of this course unit students will be able to:

- use algebraic techniques to compute the codimension of a singularity or a bifurcation;
- classify degenerate critical points;
- find a versal unfolding of a singularity;
- apply the theory of unfoldings to study bifurcations in physical systems.

Future topics requiring this course unit

Syllabus

1. **Introduction:** Families of maps. [1 Lecture]

2. **Changes of coordinates and submanifolds:** diffeomorphisms, inverse function theorem, germs, implicit function theorem, submanifolds, parametrization. Linearly adapted coordinates, transversality, Lyapunov-Schmidt reduction. [5]
3. **Some algebra:** ring of germs of smooth functions, maximal ideal, Newton diagram, finite codimension ideals. Modules. Nakayama's lemma. [3]
4. **Right equivalence:** Jacobian ideals/modules and R-tangent space, codimension, finite determinacy, splitting lemma, classification of critical points. [5]
5. **Singularities of maps:** Other singularity-theoretic equivalences, their tangent spaces, codimension and finite determinacy. [5]
6. **Unfoldings:** Families of maps as unfoldings, versal unfoldings, versality theorem for right-equivalence and contact-equivalence. [6]
7. **Applications:** Illustration of types of application, such as the geometry of curves and surfaces, gravitational catastrophe machine, ship stability, or other examples. Bifurcation of equilibria in ODEs, idea of Hopf bifurcation. [3]

Textbooks

- V. Arnold, V. Goryunov, O.V. Lyashko & V.A. Vasil'ev, Singularity Theory I. Springer. 1993.
- C.G. Gibson, Singular Points of Smooth Mappings. Pitman, 1979.
- T. Poston and I.N. Stewart, Catastrophe Theory and its Applications. Dover.
- Th. Bröcker, Differentiable Germs and Catastrophes, LMS Lecture Notes, CUP.

Learning and teaching processes

Three classes each week, of which one every fortnight is a problems class. Students should expect to spend about five hours each week on private study for this course unit.

Assessment

Coursework, weighting 10%
End of semester examination: three hours, weighting 90%

Arrangements