MATH45132 - 2012/2013

General Information

- Title: Stability Theory
- Unit code: MATH45132
- Credits: 15
- Prerequisites: MATH20401 (PDEs and vector calculus). Desirable but not essential: MATH35001 (Viscous Fluid Flow) and MATH35021 (Elasticity)
- Co-requisite units: None
- You can **not** take this course if you previously took MATH45132 *Hydrodynamic Stability Theory* as a level 4 option in year 3.
- School responsible: Mathematics
- Members of staff responsible:Dr. A. Juel

Specification

Aims

The aim of this course unit is to introduce students to the basic concepts and techniques of modern stability theory, through case studies in solid and fluid mechanics.

Brief Description of the unit

Many mechanical systems can become unstable in the sense that small disturbances superimposed on their basic state can amplify and significantly alter their initial state. In this course we introduce the basic theoretical and physical methodology required to understand and predict instability in a variety of mechanical systems subject to external forcing from buckling in solids to a variety of hydrodynamic instabilities which can lead to turbulence.

Learning Outcomes

On successful completion of this course unit students will

- derive linearised stability equations for a given basic state;
- perform a normal-mode analysis leading to an eigenvalue problems;
- use the ideas of weakly non-linear stability theory in simple systems;
- appreciate the different physical mechanisms leading to instability.

Future topics requiring this course unit

None.

Syllabus

Assuming general mechanics and fluid mechanics in particular (viscous/inviscid), as well as some aspects of dynamical systems as prerequisites for course.

1. **Introduction to stability (2 lectures)**
   Nonlinear dynamics in fluids and solids.
   Linear instability versus nonlinear instability
   Outline of the basic procedure involved in a linear stability analysis: dispersion relation, marginal stability curve
   Role of weakly nonlinear theory, e.g. normal form for pitchfork bifurcation.
   Global stability.
2. **Linear stability analysis: a case study of buckling (3 lectures)**
   Euler buckling, basic model, examples of buckling instabilities: bulk materials, auxetic materials, thin sheets.
3. **Linear stability analysis: a case study of Rayleigh-Benard convection (6 lectures)**
   Introduction to physical system, Boussinesq equations, dimensional analysis, Basic state, linear theory, normal modes,

marginal stability curve: Analytical approach for idealised boundary conditions, numerical approach (MATLAB) for generalised boundary conditions.

4. **Interfacial instabilities (2 lectures)**
   Nonlinear behaviour of interface. Rayleigh-Taylor, Saffman-Taylor and capillary instabilities.
5. **Shear flow instabilities (5 lectures)**
   Inviscid/viscous, Squire's theorem. Rayleigh's equation, Rayleigh's inflexion point criterion, Howard's semi-circle theorem, Orr-Sommerfeld equation. Examples: plane Couette flow, plane Poiseuille flow, pipe flow, Taylor-Couette flow. Problems with normal mode analysis. Pseudo-spectrum and non-normality. Absolute and convective instabilities.
6. **Effect of boundaries: convection in a finite box (4 lectures)**
   effect of finite box on linearly unstable modes, slaved modes, dimension reduction and projection onto normal modes, low dimensional dynamics.
7. **Nonlinear stability theory (5 lectures)**
   Weakly nonlinear theory, derivation of Stuart-Landau equation, Ginzburg-Landau equation
8. **Introduction to pattern formation (3 lectures)**
   Stripes, squares and hexagons, three-wave interactions, role of symmetry, long-wave instabilities of patterns: Eckhaus.
9. **Advanced topics (not scheduled): parametric instabilities**
   Faraday instability, parametric pendulum, Mathieu equations.

Textbooks

1. P.G. Drazin, Introduction to hydrodynamic stability. Cambridge University Press (2002)
2. F. Charru, Hydrodynamic Instabilities. Cambridge University Press (2011)
3. P. Manneville, Instabilities, chaos and turbulence. Imperial College Press (2004)

Teaching and learning methods

30 lectures (three lectures per week) with a fortnightly examples class. In addition should expect to spend at least seven hours each week on private study.

Assessment

Mid-semester coursework: 25%
End of semester examination: three hours weighting 75%

Arrangements