# MATH47101 - 2012/2013

### General Information

- Title: Stochastic Calculus
- Unit code: MATH47101
- Credits: 15
- Prerequisites: MATH20722 *Foundations of Modern Probability* or equivalent.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. Goran Peskir

# Specification

## Aims

The course unit aims to provide the basic knowledge necessary to pursue further studies/applications where stochastic calculus plays a fundamental role (e.g. Financial Mathematics).

## Brief description of the course unit

The stochastic integral (Itô's integral) with respect to a continuous semimartingale is introduced and its properties are studied. The fundamental theorem of stochastic calculus (Itô's formula) is proved and its utility is demonstrated by various examples. Stochastic differential equations driven by a Wiener process are studied.

## Learning outcomes

On successful completion of this course unit students will:

- understand the concept of the stochastic integral (Itô's integral);
- be able to apply Itô's formula to smooth functions of continuous semimartingales;
- know basic facts and theorems of stochastic calculus;
- understand the concept of the stochastic differential equation driven by a Wiener process.

## Future topics requiring this course unit

For MATH47112 *Brownian Motion* familiarity with the content of the present course unit is desirable but not necessary.

## Syllabus

1. The Wiener process (standard Brownian motion): Review of various constructions. Basic properties and theorems. Brownian paths are of unbounded variation. [6 lectures]
2. The Itô integral with respect to a Wiener process: Definition and basic properties. Continuous local martingales. The quadratic variation process. The Kunita-Watanabe inequality. Continuous semimartingales. The Itô integral with respect to a continuous semimartingale: Definition and basic properties. Stochastic dominated convergence theorem. [10]
3. The Itô formula: Statement and proof. Integration by parts formula. The Lévy characterization theorem. The Cameron-Martin-Girsanov theorem (change of measure). The Dambis-Dubins-Schwarz theorem (change of time). [10]
4. The Itô-Clark theorem. The martingale representation theorem. Optimal prediction of the maximum process. [4]
5. Stochastic differential equations. Examples: Brownian motion with drift, geometric Brownian motion, Bessel process, squared Bessel process, the Ornstein-Uhlenbeck process, branching diffusion, Brownian bridge. The existence and uniqueness of solutions in the case of Lipschitz coefficients. [6]

## Textbooks

- Rogers, L. C. G. and Williams, D., *Diffusions, Markov Processes and Martingales*, Vol. 1 & 2, Cambridge University Press 2000.
- Revuz, D. and Yor, M., *Continuous Martingales and Brownian Motion*, Springer 1999.

- Karatzas, I. and Shreve, S. E., *Brownian Motion and Stochastic Calculus*, Springer 1991.
- Durrett, R., *Stochastic Calculus*, CRC Press LCL 1996.

### Teaching and learning methods

Three lectures and one examples class each week. In addition students should expect to spend at least six hours each week on private study for this course unit.

### Assessment

End of semester examination: three hours weighting 100%.

## Arrangements