On-line course materials

MATH34001 - Applied Complex Analysis

Year: 3 - **Semester:** 1 - **Credit Rating:** 10

Requisites

Prerequisites
MATH20101 Real and Complex Analysis

Aims

To develop sufficient complex variable theory to introduce the complex Fourier and Laplace transforms and to apply them to the solution of partial differential equations.

Brief Description

This course unit is a natural successor to the second year course units on Complex Analysis. It introduces multivalued functions, analytic continuation and integral transforms, especially Fourier and Laplace transforms. These powerful and effective tools are used to solve many problems involving differential equations. The course is oriented towards applications rather than the theorem/proof style of development.

Learning Outcomes

On successful completion of this course unit students will be able to

- use multivalued functions of a complex variable and their applications;
- understand the notion of analytic continuation and apply it;
- use the Gamma function of complex argument;
- understand the theory of the complex Fourier and Laplace transforms and their inverses;
- use Fourier and Laplace transforms to solve a variety of linear partial differential equations with boundary and/or initial conditions.

Syllabus

- Regular Functions: Regular functions of complex z including the multivalued functions lnz and za. Branch lines and branch points. Functions with finite branch lines. [3]

- Contour Integrals: Revision of contour integrals, Cauchy's theorem, Cauchy's integral formula and the residue theorem. Evaluation of residues. Liouville's theorem. [3]
- Real Definite Integrals:. Evaluation of real definite integrals by complex contour methods, especially those involving multivalued functions of z. Deduction of new integrals from known ones by shift of contour. [4]
- Analytic Continuation:. Examples of regular functions defined by series or integrals and their analytic continuations. Uniqueness of analytic continuations and applications. Continuous continuation theorem and Schwarz's principle. [3]
- The Gamma Function: Definition of G(z) as an integral. The functional relation. Analytic continuation of G(z), its poles and residues. The reflection formula.
- Fourier and Laplace Transforms:. Integral transforms in general. Fourier's integral theorem. Functions defined on [0,), the Fourier cosine and sine transforms and their inverses. The complex Fourier transform and its inverse. Extension to the case in which the transform variable is complex and the inverse transform is a contour integral. The Laplace transform and its relationship to the complex Fourier transform. The Bromwich integral inversion formula. Examples of all of these. [4]
- Applications of Integral Transforms to Partial Differential Equations:. A simple linear ODE solved by Laplace transform. Initial value problem for the one-dimensional heat equation for the infinite bar. Same for the semi-infinite bar with appropriate end conditions. The semi-infinite bar with prescribed end temperature. Boundary value problems for Laplaces equation in an infinite strip. Same for Helmholz's equation if time permits. [5]

Teaching & Learning Process (Hours Allocated To)

Lectures	**Tutorials/ Example Classes**	**Practical Work/ Laboratory**	**Private Study**	**Total**
22	11	0	67	100

Assessment and Feedback

- Mid-semester coursework: weighting 15%
- End of semester examination: two hours weighting 85%

Further Reading

A standard source for the underlying complex variable theory is

- E.T. Copson, Functions of a Complex Variable, 1995.

For problems solved by integral transforms see

- I.N. Sneddon, The Use of Integral Transforms McGraw Hill, 1972.

Staff Involved

Dr Mike Simon - Lecturer

Data source is EPS system

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