On-line course materials

MATH10222 - Calculus and Applications A

Year: 1 - **Semester:** 2 - **Credit Rating:** 20

Requisites

Prerequisites
MATH10121 Calculus and Vectors A

Aims

1.Provide a classification of ODEs

2.Provide methods of solving both first and second-order ODEs

3.Introduce the concepts of scaling and non-dimensionalisation.

4.Introduce the concept of a regular perturbation expansion.

5.Define the main physical quantities of classical mechanics (force, velocity, acceleration, momentum etc)

6.Discuss Newton's laws of motion and gravity.

7.Introduce simple conservative and non-conservative systems involving single particles.

8.Describe basic orbital mechanics and the concept of frames of reference.

Brief Description

The unit provides a basic introduction to ordinary differential equations (ODEs) and classical mechanics. The ODE content is the first half of the course, which will discuss both methods and theory associated with general first and second order ODEs. A brief introduction to the concepts of scaling, non-dimensionalisation and regular perturbation methods will be given. In the second half of the course, the main classical-mechanics problems that motivated the development of calculus will be introduced. Basic definitions/derivations of mechanical quantities will be provided with no prior experience required/ expected. Newton's laws will be discussed and used to solve simple mechanics problems involving the motion of a single particle. Some discussion of orbital mechanics and frames of reference will be given.

The first half of the course is devoted to an introduction to the study of ordinary differential equations (ODEs). In the second half of the course the application of differential equations is illustrated by an introduction to classical mechanics.

Learning Outcomes

On successful completion of this unit students will be able to

1.Classify and first and second order ODEs.

2.Solve linear ODEs using standard methods.

3.Apply the ideas of scaling and non-dimensionalisation.

4.Apply regular perturbation methods to solve simple ODE problems.

5.Define basic mechanical quantities.

6.Solve single-particle dynamical systems (with forces of constraint).

7.Understand central-fields of force (simple orbital mechanics).

8.Solve problems relative to a non-inertial frame of reference.

Syllabus

Part 1: Ordinary differential equations (ODEs)

1.General introduction. Notation. What are ODEs? Implicit versus explicit form. Classification: order, linearity, autonomous ODEs. Boundary and initial conditions. Boundary and initial value problems. Existence and uniqueness for linear and nonlinear ODEs. [3]

2.First-order ODEs. Graphical methods; separable ODEs, ODEs of homogeneous type; integrating factor. [4]

3.Second-order ODEs. Existence and uniqueness. Linear ODEs: superposition of solutions, fundamental solutions and the general solution for homogeneous ODEs. The general solution of constant-coefficient ODEs; particular solutions for specific RHS; the method of undetermined coefficients. [If time permits (probably not): Power series expansions about regular points.] Some nonlinear ODEs with special properties (autonomous ODEs and ODEs that do not contain the dependent variable). [8]

4.Mechanics applications of second-order ODEs Damped harmonic motions of mechanical oscillators; harmonic forcing and resonance. [3]

5.Non-dimensionalisation and scaling. Exploiting small parameters in an ODE: perturbation methods. Motivation via the roots of quadratic polynomials (singular perturbations only mentioned); applications to selected (regularly perturbed) ODEs. [4]

Part 2: Mechanics

6.Introduction. Definitions, forces, moments of forces, systems in equilibrium, other coordinate systems, modelling assumptions, uniform gravitational fields. [4]

7.Newtonian dynamics. Newton's laws, Newton's second law, work and energy, motion confined to a line, the phase plane, stability of equilibrium points, motion confined to a plane, central fields of force, the path equation. [10]

8.Celestial mechanics. Kepler's `laws', properties of conics (revision not examinable), from Kepler to Newton (and back again), orbital transfer, stability of circular orbits. [4]

9.Inertial and non-inertial frames of reference. Motion relative to a moving origin, two-dimensional rotating frames, the angular frequency vector, a particle in a rotating frame. [4]

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
44	11	0	145	200

Assessment and Feedback

Assessment:

Attendance at supervisions: weighting 5%

Submission of coursework at supervisions: weighting 5%

In-class test (ODEs): weighting 5%

Take-home test (Mechanics): weighting 5%

Three hours end of semester examination: weighting 80%

Further Reading

This course is not based upon any specific textbook and all of the required material will be presented in the lectures. However, for further reading, the following references discuss the same material:

R. Bronson. Differential Equations. Schaum's Outline series.

R.D. Gregory. Classical Mechanics. Cambridge University Press.

Staff Involved

Dr Richard Hewitt - Lecturer

Prof Matthias Heil - Lecturer

Data source is EPS system
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