On-line course materials

MATH10121 - Calculus and Vectors A

Year: 1 - **Semester:** 1 - **Credit Rating:** 20

Aims

The course unit aims to provide a firm foundation in the concepts and techniques of the calculus, including real and complex numbers, standard functions, curve sketching, Taylor series, limits, continuity, differentiation, integration, vectors in two and three dimensions and the calculus of functions of more than one variable.

Brief Description

The unit introduces the basic ideas of complex numbers relating them to the standard rational and transcendental functions of calculus. The core concepts of limits, differentiation and integration are revised. Techniques for applying the calculus are developed and strongly reinforced. Vectors in two and three dimensions are introduced and this leads on to the calculus of functions of more than one variable, vector calculus, integration in the plane, Green's theorem, Stokes' theorem and Gauss' theorem.

Learning Outcomes

On successful completion of this module students will have acquired an active knowledge and understanding of the main concepts and techniques of single and multivariable calculus.

Syllabus

- Foundations: real numbers, inequalities, "infinity", intervals, absolute value or modulus, square root; imaginary numbers; complex numbers, standard form, complex plane, polar form, modulus, complex conjugate, algebra of complex numbers; functions, their graphs and basic properties, sketches of simple functions, basic idea of limits, domain and range, combining functions, simple transformations, inverse functions; trigonometric, exponential, logarithmic and hyperbolic functions and their inverses; symmetry, periodicity, increasing, decreasing, monotonicity.
- Limits and differentiation: notions of limits and continuity; discontinuities, left and right limits; limits of sums, products, quotients, compositions, l'Hpital's rule; order notation; definition of derivative and higher derivatives; derivatives of inverse functions; derivatives of trigonometric functions, exponential, logarithm, powers, hyperbolic functions and their inverse functions; sums, products, quotients and chain rule; derivatives of implicit and parametric functions; logarithmic differentiation; turning and critical points; curves at right angles.
- Power series: notion of a power series as a limit of polynomials, its derivative and integral; Taylor series; radius of convergence (not proven); series for exponential, sine, cosine and other functions; manipulating Taylor expansions.

- More on complex numbers: Euler's formula; complex forms of sine and cosine, relationship between trigonometric and hyperbolic identities; de Moivre's theorem; exponential and polar forms of complex numbers; complex roots.
- Integration: definite and indefinite integrals; fundamental theorem of calculus; proper and improper integrals; singularities in definite integrals; techniques for integration: standard integrals, constant factor, function of a linear function, integral of a sum, algebraic manipulation, integration by parts, recurrence formulae, substitutions (and their identification), substitution in definite integrals, partial fractions; area between curves (including polar coordinates), lengths of curves (including polar coordinates), surfaces and volumes of revolution.
- Curve sketching: Cartesian and polar coordinates; simple examples; parametric curves; generic conic sections; powers (with positive and negative integer exponent); factorised polynomials and rational functions, zeros, singularities, asymptotic behaviour, turning or critical points; simple curves in polar coordinates.
- Vectors in two and three dimensions: points and vectors; vectors as directed line segments (magnitude, direction); Cartesian representation, coordinates; basic properties, addition, subtraction, multiplication by a scalar, magnitude, unit vectors (and the unit vectors i, j and k), inner or "dot" product, angle between two vectors, orthogonality, components or projections, extension to higher dimensions; vector or "cross" product in three dimensions, its magnitude and direction (orthogonality), cross product as a "turning effect"; scalar triple product; vector representation of points, lines and planes, and determining relations between them; other orthogonal coordinate systems (including polar, parabolic, elliptic, cylindrical and spherical).
- Functions of more than one variable: partial derivative (definition, notation and evaluation), chain-rule, higher partial derivatives including mixed derivatives, higher dimensional Taylor expansion; gradient vector ("grad"); critical or turning points (maxima, minima, saddle-points), relationship with contour lines and "grad", identifying critical points, derivative test and use of the discriminant; turning points with constraints, Lagrange multipliers; "grad", "div", "curl" and some useful identities in vector calculus.
- Multiple integrals: area integration of a scalar-valued function in the plane, choice of order of integration; area integral in polar coordinates; Jacobian and change of variable; directed path or line integral of a vector valued function in the plane, path-dependence, path independence, conservative vector fields and potential functions; Green's and Stokes' theorems in the plane (relating area and line integrals), divergence theorem or Gauss' theorem; the meaning of Green's, Stokes' and Gauss' theorems.

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
44	11	0	145	200

Assessment and Feedback

Supervision attendance and participation; Weighting within unit 10%

Coursework; In-class test in week 6, weighting within unit 15%

Three hours end of semester examination; Weighting within unit 75%

Further Reading

James Stewart, Calculus, Early Transcendentals, International Student Edition, Thomson (any recent edition).

[This text covers almost every aspect of what you will be learning, with many examples. You should ensure that you can have easy access to a copy.

Useful background material can be found in:

Hugh Neill and Douglas Quadling. Cambridge Advanced Mathematics Core 3 & 4.

[This text describes well and clearly what should be known from A-level. It (as for other A-level texts) provides an introduction to what should be known before entering a university calculus course.]

Staff Involved

Prof Oliver Jensen - Lecturer

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