

On-line course materials

MATH45061 - Continuum Mechanics

Year: 4 - Semester: 1 - Credit Rating: 15

Requisites

Prerequisites

MATH20401 Partial Differential Equations and Vector Calculus A

MATH35001 Viscous Fluid Flow

MATH35021 Elasticity

MATH35001, Viscous Fluid Flow, and MATH35021, Elasticity, are desirable, but not essential.

Aims

The course unit concerns the formulation and solution of problems in continuum mechanics (solid and fluid mechanics) from a modern unified perspective. The aims are (i) to introduce students to the general analytic machinery of tensor calculus, variational principles and conservation laws in order to formulate governing equations; and (ii) to be aware of exact, approximate and numerical methods to solve the resulting equations.

Brief Description

This unit describes the fundamental theory of continuum mechanics in a unified mathematical framework. The unit will cover theories of nonlinear and linear elasticity together with those of compressible and incompressible fluid mechanics.

Learning Outcomes

On successful completion of this course unit students will

- Be able to formulate governing equations for a variety of problems in continuum mechanics.
- Understand the relationship between the general theory and its specialisation to the equations of linear elasticity and incompressible Newtonian fluid mechanics.
- Solve simple problems in continuum mechanics analytically.
- Be aware of certain numerical techniques that can be applied to problems in continuum mechanics.

Syllabus

- Foundations and Fundamentals [3]. Eulerian and Lagrangian coordinates, vectors and tensors, material/convected derivatives, integral theorems
- Kinematics: Deformation and Flow [3]. Measures of strain, stretch, rotation, compatibility, applications of convective derivative, velocity and vorticity.
- Stress [3]. The continuum hypothesis, Cauchy's stress principle, measures of stress, objectivity.
- Fundamental laws, governing equations and thermodynamics [4]. Conservation laws (mass, linear and angular momentum, energy), Cauchy's equations, variational principles, thermodynamics, constitutive modelling.
- Linear elasticity [3]. Hooke's law, Navier-Lame equations, elastostatics, elastodynamics, Airy stress function.
- Incompressible Newtonian Fluids [3]. Viscosity, potential flow, slow flow, high-speed flow, boundary layers
- Nonlinear elasticity [4]. Hyperelasticity and constitutive models, simple exact solutions, principle of virtual work and basic finite elements.
- Complex fluids [4]. Non-Newtonian behaviour, shear thinning/thickening, viscoelasticity.
- Further advanced topics[3]. Interfaces and Fluid-structure interaction.

Teaching & Learning Process (Hours Allocated To)

| Lectures | Tutorials/ Example Classes | Practical Work/ Laboratory | Private Study | Total |
|-----------------|---|---|----------------------|--------------|
| 22 | 11 | 0 | 117 | 150 |

Assessment and Feedback

- Mid-semester coursework: 25%
- End of semester examination: three hours weighting 75%

Further Reading

- Spencer, A.J.M, "Continuum Mechanics", Dover
- Gonzalez, O. and Stuart, A.M., "A first course in continuum mechanics", CUP
- Irgens, F., "Continuum Mechanics", Springer

Staff Involved

Dr Andrew Hazel - Lecturer