On-line course materials

MATH31061 - Differentiable Manifolds

Year: 3 - **Semester:** 1 - **Credit Rating:** 10

Aims

The unit aims to introduce the basic ideas of differentiable manifolds.

Brief Description

Differentiable manifolds are among the most fundamental notions of modern mathematics. Roughly, they are geometrical objects that can be endowed with coordinates; using these coordinates one can apply differential and integral calculus, but the results are coordinate-independent.

Examples of manifolds start with open domains in Euclidean space Rn , and include "multidimensional surfaces" such as the n-sphere Sn and n-torus Tn , the projective spaces RPn and CPn , and their generalizations, matrix groups such as the rotation group SO(n), etc. Differentiable manifolds naturally appear in various applications, e.g., as configuration spaces in mechanics. They are arguably the most general objects on which calculus can be developed. On the other hand, differentiable manifolds provide for calculus a powerful invariant geometric language, which is used in almost all areas of mathematics and its applications.

In this course we give an introduction to the theory of manifolds, including their definition and examples; vector fields and differential forms; integration on manifolds and de Rham cohomology.

Learning Outcomes

On completion of this unit successful students will be able to:

- deal with various examples of differentiable manifolds and smooth maps;
- have familiarity with tangent vectors, tensors and differential forms;
- work practically with vector fields and differential forms;
- appreciate the basic ideas of de Rham cohomology and its examples;
- apply the ideas of differentiable manifolds to other areas.

Syllabus

- Manifolds and smooth maps. Coordinates on familiar spaces. Charts and atlases. Definitions of manifolds and smooth maps. Products. Specifying manifolds by equations. More examples of manifolds.
- Tangent vectors. Velocity of a curve. Tangent vectors. Tangent bundle. Differential of a map.

- Topology of a manifold. Topology induced by manifold structure. Identification of tangent vectors with derivations. Bump functions and partitions of unity. Embedding manifolds in RN.
- Tensor algebra. Dual space, covectors and tensors. Einstein notation. Behaviour under maps. Tensors at a point. Example: differential of a function as covector.
- Vector fields. Tensor and vector fields. Examples. Vector fields as derivations. Flow of a vector field. Commutator.
- Differential forms. Antisymmetric tensors. Exterior multiplication. Forms at a point. Bases and dimensions. Exterior differential: definition and properties.
- Integration. Orientation. Integral over a compact oriented manifold. Independence of atlas and partition of unity. Integration over singular manifolds and chains. Stokes theorem.
- De Rham cohomology. Definition of cohomology and examples of nonzero classes. Poincar Lemma. Examples of calculation.

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
22	11	0	67	100

Assessment and Feedback

- Coursework; Weighting within unit 20%
- 2 hours end of semester examination; Weighting within unit 80%

Further Reading

No particular textbook is followed. Students are advised to keep their own lecture notes. There are many good sources available treating various aspects of differentiable manifolds on various levels and from different viewpoints. Below is a list of texts that may be useful. More can be found by searching library shelves.

- R. Abraham, J. E. Marsden, T. Ratiu. Manifolds, tensor analysis, and applications.
- B.A. Dubrovin, A.T. Fomenko, S.P. Novikov. Modern geometry, methods and applications.
- A. S. Mishchenko, A. T. Fomenko. A course of differential geometry and topology.
- S. Morita. Geometry of differential forms.
- Michael Spivak. Calculus on manifolds.
- Frank W. Warner. Foundations of differentiable manifolds and Lie groups.

Staff Involved

Dr Ted Voronov - Lecturer

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