On-line course materials

MATH41112 - Ergodic Theory (Reading Course)

Year: 4 - **Semester:** 2 - **Credit Rating:** 15

Requisites

Prerequisites
MATH20122 Metric Spaces

Aims

To obtain an understanding and appreciation of the complexity of the orbit structure of chaotic dynamical systems. To work comfortably with invariant measures and ergodic measures. To apply these ideas to a number of relevant examples, with particular reference to Birkhoff's Ergodic Theorem.

Brief Description

Dynamical systems is the study of iterating a given map. That is, we take X to be some mathematical space (for example, an interval, a circle, or perhaps something more complicated) and a map T from X to itself. We then take a point x in X and repeatedly apply T, obtaining the sequence of points {x, T(x), T (T(x)), ...}; this is called the orbit of x.

These orbits are generally very complicated. For example, two points x and y that start very close to each other may have very different orbits; this is known as sensitive dependence on initial conditions and is the one of the motivations for what has popularly become known as Chaos Theory.

A general dynamical system may be so chaotic that it is impossible to describe every orbit. Instead, we could attempt to describe what a typical orbit looks like; this is the basis of Ergodic Theory. To make `typical' precise, we need to use measure theory, and a self-contained introduction to this will be given.

We will see that ergodic theory allows us to prove several interesting and surprising results in other areas of mathematics, particularly in number theory. Here is one example: Consider the sequence 1,2,4,8,16,32,...,2n, and consider the sequence of leftmost (or leading)

digits: 1,2,4,8,1,3,... How often does the digit 7, say, appear in this sequence? We will use ergodic theory to prove that about 5.8% of the digits in the above sequence are 7s (the precise answer is log 8/7).

One highlight of the course is Birkhoff's Ergodic Theorem. This beautiful theorem says that (under appropriate hypotheses!) the proportion of time that a typical orbit spends in some region of X is equal to the measure (area/volume) of the region. We shall then apply this result to our examples, deriving some interesting and useful corollaries.

Learning Outcomes

On successful completion of the course unit students will be able to:

- understand the different kinds of orbits that may arise in the study of dynamical system;
- understand the basic concepts in ergodic theory, such as measure theory, uniform distribution, invariant measures, ergodicity;
- describe the asymptotic behaviour of ergodic averages via Birkhoff's Ergodic Theorem;
- apply ergodic theory to a number of fundamental examples, rotations on tori, the doubling map, toral automorphisms, the continued fraction map, Bernoulli shifts and Markov shifts.

Future topics requiring this course unit

None

Syllabus

- Introduction
- Uniform distribution mod 1
- Examples of dynamical systems
- An introduction to measure theory
- Invariant and ergodic measures
- Topological dynamics
- Birkhoff's ergodic theorem and applications

Teaching & Learning Process (Hours Allocated To)

Lectures	**Tutorials/ Example Classes**	**Practical Work/ Laboratory**	**Private Study**	**Total**
11	0	0	139	150

Assessment and Feedback

- End of semester examination (3 hours) 100%.

Further Reading

Good books on ergodic theory include

- P. Walters, An Introduction to Ergodic Theory, Springer-Verlag, 1981,
- W. Parry, Topics in Ergodic Theory, Cambridge, 1981.

Our approach to Ergodic Theory is most closely related to that in Walters' book, although both books contain far more material than is in the course.

Staff Involved

Dr Charles Walkden - Lecturer

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