On-line course materials

MATH20722 - Foundations of Modern Probability

Year: 2 - **Semester:** 2 - **Credit Rating:** 10

Requisites

Prerequisites
MATH10141 Probability 1
MATH20701 Probability 2

Aims

The course unit aims to

- provide the basic knowledge of facts and methods needed to state and prove the law of large numbers and the central limit theorem;
- introduce fundamental concepts and tools needed for the rigorous understanding of third and fourth level course units on probability and stochastic processes including their applications (e.g. Financial Mathematics).

Brief Description

The law of large numbers and the central limit theorem are formulated and proved. These two results embody the most important results of classical probability theory having an endless number of applications. A good understanding of MATH10101 or 10111, 20101 or 20111, and 20701 is required.

Learning Outcomes

On completion of this unit successful students will:

- understand the meaning and proof of the law of large numbers;
- understand the meaning and proof of the central limit theorem;
- be able to apply the methods of proof to related problems.

Future units for which this unit is desirable but not essential.

MATH37001 Martingales with Applications to Finance (level 3 semester 1)

MATH47101 Stochastic Calculus (level 4 semester 1)

MATH47112 Brownian Motion (level 4 semester 2)

Syllabus

1.Probability measures. Probability spaces. Random variables. Random vectors. Distribution functions. Density functions. Laws. The two Borel-Cantelli lemmas. The Kolmogorov 0-1 law. [4 lectures]

2.Inequalities (Markov, Jensen, Hlder, Minkowski). Modes of convergence (almost sure, in probability, in distribution, in mean). Convergence relationships. [4]

3.Expectation of a random variable. Expectation and independence. The Cesro lemma. The Kronecker lemma. The law of large numbers (weak and strong). [5]

4.Fourier transforms (characteristic functions). Laplace transforms. Uniqueness theorems for Fourier and Laplace transforms. Convergence of characteristic functions: the continuity theorem. The central limit theorem. [6]

5.Brownian motion as the weak limit of the random walk. Donsker's Theorem. [3]

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
22	11	0	67	100

Assessment and Feedback

- Mid-semester coursework: weighting 20%
- End of semester examination: two hours weighting 80%

Further Reading

- D. Williams, Probability with Martingales, Cambridge University Press 1991.
- A.N. Shiryaev, Probability, Springer-Verlag 1996.
- G.R. Grimmett and D.R. Stirzaker, Probability and Random Processes, Oxford University Press 1992.

Staff Involved

Dr Jonathan Bagley - Lecturer

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