On-line course materials

MATH31011 - Fourier Analysis and Lebesgue Integration

Year: 3 - **Semester:** 1 - **Credit Rating:** 10

Requisites

Prerequisites
MATH20101 Real and Complex Analysis
MATH20111 Real Analysis

Aims

To give an introduction to Lebesgue's theory of integration on the set of real numbers R. To use this to find an appropriate setting in which to understand the convergence of Fourier series.

Brief Description

It is often convenient to represent functions as Fourier series. However, the convergence of such series is a delicate issue closely related to the theory of integration. A standard approach to integration on the real line, formalised by Riemann, is based on partitioning the domain into smaller intervals. This approach works in many situations but there are simple examples for which it fails. In the early 1900s, H. Lebesgue produced a better theory in which the key idea is to extend the notion of length from intervals to more complicated subsets of R. This started an area of mathematics it its own right, called Measure Theory. Most generally, this is about how one may sensibly assign a size to members of a collection of sets. One application of Lebesgue's ideas is that one can introduce a vector space of functions in which Fourier series appear in a natural way.

This course will appeal to students who have enjoyed MATH20101 or MATH20111 and MATH20122. It will be useful to student taking probability courses in years three and four since the ideas of measure theory have a central role in probability theory.

Learning Outcomes

On successful completion of this course unit students will

- understand how Lebesgue measure on R is defined,
- understand how measures may be used to construct integrals,

- know the basic convergence theorems for the Lebesgue integral,
- understand the relation between Fourier series and the Hilbert space of square integrable functions.

Syllabus

- Fourier series, convergence and Dirichlet's Theorem. [3 lectures]
- Revision of countable and uncountable sets, the Cantor set. [2 lectures]
- Riemann's approach to integration. Lesbesgue measure on R, Borel sets, measurable sets and functions, construction of the Lebesgue integral. [8 lectures]
- Limit theorems for the Lebesgue integral. [3 lectures]
- Square integrable functions and Fourier series, Hilbert spaces. [6 lectures]

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
22	11	0	67	100

Assessment and Feedback

- Mid-semester coursework: weighting 15%
- End of semester examination: two hours weighting 85%

Further Reading

- J. Franks, A (Terse) Introduction to Lebesgue Integration, American Mathematical Society, Student Mathematical Library, 2009.
- H. S. Bear, A Primer of Lebesgue Integration, Academic Press, 1995.

Staff Involved

Dr Alena Vencovska - Lecturer

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