On-line course materials

MATH34032 - Greens Functions, Integral Equations and Applications

Year: 3 - **Semester:** 2 - **Credit Rating:** 10

Requisites

Prerequisites
MATH20401 or MATH20411

Aims

To introduce students to Green's functions and integral equations (and how they are linked). To explain how Green's functions may be used in practice, with applications to a variety of ordinary and partial differential equations and physical applied problems such as potential flow, wave propagation and scattering.

Brief Description

This is a methods based course, focusing on the theory and application of Green's functions in applied mathematics. Green's functions enable the solution to a variety of interesting and important problems. In particular one can set up solutions to ordinary and partial differential equations of general type in integral form by the use of a Green's function. In more difficult problems, such as scattering from an object, integral equations result. In this course we will show how Green's functions are defined, why they are important and then show their application to various problems in applied mathematics. In particular we will exhibit how they may be used to understand wave propagation on a string, potential flow on bounded domains, wave propagation and scattering from an object and how one can construct an acoustic "cloak" in order to render regions invisible from acoustic waves (and thus construct theoretical "domains of silence").

Learning Outcomes

On successful completion of this course students will:

- Have acquired sound knowledge of Green's functions and Fredholm and Volterra integral equations
- Have solved representative problems in applied mathematics using the above, eg solving ordinary and partial differential equations using Green's functions, obtaining and solving linear integral equations.

Syllabus

- Section 1: Preliminaries. Dirac Delta function, Heaviside function, Operators, Adjoint operator [1 lecture]
- Section 2: Greens functions in 1D. Construction for constant coefficient ODEs and Sturm Liouville problems. Applications to the steady state heat equation and wave equation. [5 lectures]
- Section 3: Greens functions in 2 and 3D. Steady state heat equation and Potential flow problems (Laplace) and time-harmonic wave equation (Helmholtz). Applications to cloaking. [5 lectures]
- Section 4: Integral equations in 1D. Motivated by 1D scattering problem. Series solution and physical interpretation. General integral equation types. Degenerate (separable) kernels and solution method. Neumann series and iterated kernels. [4 lectures]
- Section 5: Integral equations in 2 and 3D. Greens second identity. Generation of integral equation for Potential Flow problems (Laplace) via Greens functions for bounded domains. Single and double layer potentials. Solution via Boundary Element methods. Extention to integral equation for inhomogeneity in steady state thermal problem and Potential flow. Eshelbys conjecture. Applications to homogenization. [7 lectures]

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
22	11	0	67	100

Assessment and Feedback

- Coursework: Test in week 8 or 9 worth 20%
- End of semester 2 hour Examination 80%

Further Reading

- GF Roach, Green's functions, introductory theory with applications, Van Nostrand Reinhold, 1982
- I Stakgold, MJ Holst, Green's Functions and Boundary Value Problems, John Wiley and Sons, 2011
- D Porter and DSG Stirling, Integral Equations: A Practical Treatment, Cambridge University Press, 1990
- E Zauderer, Partial Differential Equations of Applied Mathematics, John Wiley and Sons, 1983

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