

On-line course materials

MATH42001 - Group Theory

Year: 4 - Semester: 1 - Credit Rating: 15

Aims

This lecture course unit aims to introduce students to some more sophisticated concepts and results of group theory as an essential part of general mathematical culture and as a basis for further study of more advanced mathematics.

Brief Description

The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outstanding beauty. It takes just three simple axioms to define a group, and it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts and results of Group Theory.

Learning Outcomes

On successful completion of this course unit students will have acquired

- a sound understanding of the classification of finitely generated abelian groups,
- knowledge of some fundamental results and techniques from the theory of finite groups,
- knowledge of group actions on sets, simple groups, Sylow's theorems and various applications of Sylow's theorems.

Syllabus

- Revision of basic notions (subgroups and factor groups, homomorphisms and isomorphisms), generating sets, commutator subgroups. [2 lectures]
- Abelian groups, the Fundamental Theorem on finitely generated abelian groups. [4]
- The Isomorphism Theorems. [3]
- Simple groups, the simplicity of the alternating groups. [3]
- Group actions on sets, orbits, stabilizers, the number of elements in an orbit, Burnside's formula for the number of orbits, conjugation actions, centralizers and normalizers. [5]
- Sylow's Theorems, groups of order pq , pqr . [3]

The lectures will be enhanced by additional reading on related topics:

- commutators and some of their elementary properties;
- proof of Ito's theorem which states that a group which is the product of two abelian groups has trivial second derived subgroup;
- finite abelian groups, including a proof that the rank and torsion coefficients determine the abelian group uniquely;
- proving that A_n for n greater than or equal to 5 is a simple group;
- proving that a simple group of order 60 must be isomorphic to A_5 .

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
22	11	0	117	150

Assessment and Feedback

- Coursework: in-class test weighting 7%
- End of semester examination: three hours, weighting 93%

Further Reading

Recommended text:

- John B Fraleigh, A First Course in Abstract Algebra, (5th edition), 1967, Addison-Wesley.

Staff Involved

Prof Peter Rowley - Lecturer

Data source is EPS system

Back To Top