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On-line course materials

MATH42041 - Noncommutative Algebra

Year: 4 - **Semester:** 1 - **Credit Rating:** 15

Aims

To introduce students to noncommutative algebra.

Brief Description

Nature is inherently noncommutative---just try putting on your shoes and socks in the wrong order---and noncommutative structures are increasingly important throughout mathematics and physics. In this course, we will examine in detail some of the most important noncommutative algebras that appear "in nature" and prove some of the basic structure theorems about noncommutative rings. One of the most fundamental such algebras is the quaternions---if you ignore the fact that it is not commutative, then this is a field that is 4-dimensional as a real vector space! Famously when Hamilton discovered it in the 19-th century he carved its formulae on a bridge lest he forget them. Another is the Weyl algebra---sometimes called the "algebra of quantum mechanics" since its structure encodes the Uncertainty Principle, and sometimes called a ring of differential operators since it encodes the algebraic aspects of differential equations.

Learning Outcomes

On successful completion of this course unit students will

- have a deepened understanding of algebra and in particular of module theory;
- understand fundamental noncommutative algebras like the quaternions and the Weyl algebra (or algebra of quantum mechanics);
- understand some of the basic structure theorems in noncommutative algebra, like the Wedderburn density theorem, the classification of simple Artinian rings and the structure of modules over principal ideal domains.

Syllabus

- Introduction and preliminaries: fields, rings and matrices;
- Examples of noncommutative rings, the quaternions and the Weyl algebra;
- Wedderburn-Artin theory - many rings are matrices;
- The structure of Artinian rings;
- Introduction to representation theory;
- The structure of modules over principal ideal domains;
- Division rings.

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
33	0	0	117	150

Assessment and Feedback

- Coursework: weighting 20%
- End of semester examination: two and a half hours weighting 80%

Further Reading

- Cohn, P. M. Introduction to ring theory. Springer-Verlag, 2000.
- Passman, D. S. A course in ring theory. The Wadsworth & Brooks/Cole Mathematics Series, 1991.
- Lam, T. Y. A first course in noncommutative rings. Second edition. Graduate Texts in Mathematics, 131. Springer-Verlag, New York, 2001

Staff Involved

Prof Toby Stafford - Lecturer

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