On-line course materials

MATH20411 - Partial Differential Equations and Vector Calculus B

Year: 2 - **Semester:** 1 - **Credit Rating:** 10

Requisites

Prerequisites
MATH10121 Calculus and Vectors A
MATH10131 Calculus and Vectors B

Aims

This course introduces students to analytical and numerical methods for solving partial differential equations (PDEs) and builds on the first year core applied mathematics courses to develop more advanced ideas in differential and integral calculus.

Brief Description

The main topics to be explored are: Fourier series, partial differential equations, analytical and numerical methods for solving classical PDEs (Laplace's equation and the heat and wave equations) and several topics in vector calculus, including surface and volume integrals. The course covers similar material to MATH20401 but contains a reduced range of topics and with fewer details, where appropriate. The methods employed in the course will prove essential for all of the applied mathematics and numerical analysis options in the remaining semesters of the Joint Honours BSc and MMath degree programmes.

Learning Outcomes

On completion of this unit successful students will be able to:

- convert Cartesian coordinates into cylindrical and spherical coordinates and sketch surfaces expressed in all of these coordinate systems;
- perform partial differentiation of functions of more than one variable and interpret these derivatives physically;
- recognise classical PDEs describing physical processes such as heat diffusion;
- interpret boundary and initial conditions physically;
- understand the basic concept of orthogonal functions;

- compute Fourier series, Fourier sine series and Fourier cosine series of some piecewise continuous functions;
- classify second-order PDEs as being either elliptic, hyperbolic or parabolic;
- solve, analytically, via the method of separation of variables, the heat and wave equations (in one space variable) and Laplace's equation (in two space variables), on rectangular and circular domains;
- solve, numerically, via finite difference schemes, the heat equation in one space variable;
- solve, numerically, convection-diffusion equations using upwind finite differencing;
- compute elements of surface and volume in different coordinate systems;
- evaluate line, surface and volume integrals over a range of domains, using transformations to other coordinate systems where appropriate;
- understand grad, div and curl operator notation and relate some key identities to physical properties of vector fields;
- understand, and interpret physically, the classical Divergence, Green's and Stokes' theorems.

Syllabus

- Introductory material. [3 lectures]
- Cartesian, cylindrical and spherical coordinates. Functions of several variables, surfaces. Partial derivatives, chain rule. Partial differential equations, boundary and initial conditions. Integrals of functions of several variables.
- Fourier series. [4 lectures]
- Orthogonality. Fourier series and Fourier coefficients. Periodic, even and odd functions. Fourier's theorem. Fourier sine and cosine series.
- Partial Differential Equations. [2 lectures]
- Linearity, homogeneity and order of PDEs. Classification of second-order equations. Introduction to the classical equations: Laplace's, heat and wave equations.
- Analytical Solution of PDEs. [4 lectures]
- The method of separation of variables. Solving, exactly, initial-value problems for the heat and wave equations. Eigenfunction series and normal modes. Solving Laplace's equation in both Cartesian and plane polar co-ordinates. Applications to heat conduction and electrostatics.
- Numerical Solution of PDEs. [4 lectures]
- Solving, approximately, the reaction-diffusion and convection-diffusion equations (ODEs) via finite difference methods.
- Solving, approximately, the heat equation in one space variable (PDE). Explicit and implicit numerical schemes.
- Vector Calculus. [5 lectures]
- Surfaces, unit vectors, elements of surface/volume. Line, surface and volume integrals. Scalar and vector fields: differential and integral calculus. Grad, div and curl operators and related identities. Classical theorems: Divergence, Green's and Stokes' theorems.

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
22	11	0	67	100

Assessment and Feedback

- Coursework: an in-class test, weighting within unit 20%
- 2 hour end of semester examination: weighting within unit 80%

Further Reading

- Morton, K.W., Mayers, D.F, Numerical solution of partial differential equations, Cambridge University Press, 2005.
- James Stewart, Calculus, Early Transcendentals, Thomson, fifth edition (international student edition), 2003.
- R Haberman, Elementary Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, (Third edition) Prentice-Hall, 1998.
- Schey, H. M. Div, Grad, Curl, and all that : an Informal Text on Vector Calculus, New York : W. W. Norton, various editions.

Staff Involved

Dr Catherine Powell - Lecturer

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