On-line course materials

MATH45132 - Stability Theory

Year: 4 - **Semester:** 2 - **Credit Rating:** 15

Requisites

Prerequisites
MATH20401 Partial Differential Equations and Vector Calculus A
Desirable but not essential: MATH35001 (Viscous Fluid Flow) and MATH35021 (Elasticity)

Aims

The aim of this course unit is to introduce students to the basic concepts and techniques of modern stability theory, through case studies in solid and fluid mechanics.

Brief Description

Many mechanical systems can become unstable in the sense that small disturbances superimposed on their basic state can amplify and significantly alter their initial state. In this course we introduce the basic theoretical and physical methodology required to understand and predict instability in a variety of mechanical systems subject to external forcing from buckling in solids to a variety of hydrodynamic instabilities which can lead to turbulence.

Learning Outcomes

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On successful completion of this course unit students will

- derive linearised stability equations for a given basic state;
- perform a normal-mode analysis leading to an eigenvalue problems;
- use the ideas of weakly non-linear stability theory in simple systems;
- appreciate the different physical mechanisms leading to instability.

Future topics requiring this course unit

None.

Syllabus

Assuming general mechanics and fluid mechanics in particular (viscous/inviscid), as well as some aspects of dynamical systems as prerequisites for course.

1. Introduction to stability (3 lectures)

Nonlinear dynamics in fluids and solids. Linear instability versus nonlinear instability. Outline of the basic procedure involved in a linear stability analysis: dispersion relation, marginal stability curve. Role of weakly nonlinear theory, e.g. normal form for pitchfork bifurcation. Global stability.

2. Linear stability analysis: a case study of Rayleigh-Benard convection (5 lectures)

Introduction to physical system, Boussinesq equations, dimensional analysis, Basic state, linear theory, normal modes, marginal stability curve: Analytical approach for idealised boundary conditions, numerical approach (MATLAB) for generalised boundary conditions.

3. Interfacial instabilities (3 lectures)

Nonlinear behaviour of interface. Rayleigh-Taylor, Saffman-Taylor and capillary instabilities.

4. Shear flow instabilities (5 lectures)

Inviscid/viscous, Squire's theorem. Rayleigh's equation, Rayleigh's inflexion point criterion, Howard's semi-circle theorem, Orr-Sommerfeld equation. Examples: plane Couette flow, plane Poiseuille flow, pipe flow, Taylor-Couette flow. Problems with normal mode analysis. Pseudo-spectrum and non-normality. Absolute and convective instabilities.

5. Instabilities in solid: Euler buckling (2 lectures)

Euler buckling, basic model, examples of buckling instabilities: bulk materials, auxetic materials, thin sheets.

6. Bifurcation theory (3 lectures)

Local bifurcations, normal forms, structural stability, Lorenz model, route to chaos.

7. Nonlinear stability theory (6 lectures)

Weakly nonlinear theory, derivation of Stuart-Landau equation, Ginzburg-Landau equation

8.Introduction to pattern formation (if time allows)

Stripes, squares and hexagons, three-wave interactions, role of symmetry, long-wave instabilities of patterns: Eckhaus.

9. Advanced topics (by examples only): parametric instabilities

Faraday instability, parametric pendulum, Mathieu equations.

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total

30	6	0	114	150

Assessment and Feedback

- Mid-semester coursework: 25%
- End of semester examination: two and a half hours weighting 75%

Further Reading

- P.G. Drazin, Introduction to hydrodynamic stability. Cambridge University Press (2002)
- F. Charru, Hydrodynamic Instabilities. Cambridge University Press (2011)
- P. Manneville, Instabilities, chaos and turbulence. Imperial College Press (2004)

Staff Involved

Prof Anne Juel - Lecturer

Data source is EPS system

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