On-line course materials

MATH47101 - Stochastic Calculus

Year: 4 - **Semester:** 1 - **Credit Rating:** 15

Aims

The course unit aims to provide the basic knowledge necessary to pursue further studies/applications where stochastic calculus plays a fundamental role (e.g. Financial Mathematics).

Brief Description

The stochastic integral (Ito's integral) with respect to a continuous semimartingale is introduced and its properties are studied. The fundamental theorem of stochastic calculus (Ito's formula) is proved and its utility is demonstrated by various examples. Stochastic differential equations driven by a Wiener process are studied.

Learning Outcomes

On successful completion of this course unit students will:

- understand the concept of the stochastic integral (Ito's integral);
- be able to apply Ito's formula to smooth functions of continuous semimartingales;
- know basic facts and theorems of stochastic calculus;
- understand the concept of the stochastic differential equation driven by a Wiener process.

Syllabus

- The Wiener process (standard Brownian motion): Review of various constructions. Basic properties and theorems. Brownian paths are of unbounded variation. [6 lectures]
- The Ito's integral with respect to a Wiener process: Definition and basic properties. Continuous local martingales. The quadratic variation process. The Kunita-Watanabe inequality. Continuous semimartingales. The Ito's integral with respect to a continuous semimartingale: Definition and basic properties. Stochastic dominated convergence theorem. [10]
- The Ito's formula: Statement and proof. Integration by parts formula. The Levy characterization theorem. The Cameron-Martin-Girsanov theorem (change of measure). The Dambis-Dubins-Schwarz theorem (change of time). [10]
- The Ito's-Clark theorem. The martingale representation theorem. Optimal prediction of the maximum process. [4]
- Stochastic differential equations. Examples: Brownian motion with drift, geometric Brownian motion, Bessel process, squared Bessel process, the Ornstein-Uhlenbeck process, branching diffusion, Brownian bridge. The existence and uniqueness of solutions in the case of Lipschitz coefficients. [6]

Teaching & Learning Process (Hours Allocated To)

Lectures	Tutorials/ Example Classes	Practical Work/ Laboratory	Private Study	Total
33	11	0	106	150

Assessment and Feedback

End of semester examination: three hours weighting 100%

Further Reading

- Rogers, L. C. G. and Williams, D., Diffusions, Markov Processes and Martingales, Vol. 1 & 2, Cambridge University Press 2000.
- Revuz, D. and Yor, M., Continuous Martingales and Brownian Motion, Springer 1999.
- Karatzas, I. and Shreve, S. E., Brownian Motion and Stochastic Calculus, Springer 1991.
- Durrett, R., Stochastic Calculus, CRC Press LCL 1996.

Staff Involved

Prof Goran Peskir - Lecturer

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