MATH34001

Applied Complex Analysis

Unit code:	MATH34001
Credit Rating:	10
Unit level:	Level 3
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20101 - Real and Complex Analysis (Compulsory)
- MATH20111 - Real Analysis (Compulsory)
- MATH20401 - Partial Differential Equations and Vector Calculus A (Compulsory)
- MATH20411 - Partial Differential Equations and Vector Calculus B (Compulsory)

Additional Requirements

MATH34001 pre-requisites

Students must have taken (MATH20101 OR MATH20111) AND (MATH20401 OR MATH20411)

Aims

To develop sufficient complex variable theory to introduce the complex Fourier and Laplace transforms and to apply them to the solution of partial differential equations.

Overview

This course unit is a natural successor to the second year course units on Complex Analysis. It introduces multivalued functions, analytic continuation and integral transforms, especially Fourier and Laplace transforms. These powerful and effective tools are used to solve many problems involving differential equations. The course is oriented towards applications rather than the theorem/proof style of development.

Assessment methods

- Other - 15%
- Written exam - 85%

Assessment Further Information

- Mid-semester coursework: weighting 15%
- End of semester examination: two hours weighting 85%

Learning outcomes

On successful completion of this course unit students will be able to

- use multivalued functions of a complex variable and their applications;
- understand the notion of analytic continuation and apply it;
- use the Gamma function of complex argument;
- understand the theory of the complex Fourier and Laplace transforms and their inverses;
- use Fourier and Laplace transforms to solve a variety of linear partial differential equations with boundary and/or initial conditions.

Syllabus

- Regular Functions: Regular functions of complex z including the multivalued functions lnz and za. Branch lines and branch points. Functions with finite branch lines. [3]
- Contour Integrals: Revision of contour integrals, Cauchy's theorem, Cauchy's integral formula and the residue theorem. Evaluation of residues. Liouville's theorem. [3]
- Real Definite Integrals:. Evaluation of real definite integrals by complex contour methods, especially those involving multivalued functions of z. Deduction of new integrals from known ones by shift of contour. [4]
- Analytic Continuation:. Examples of regular functions defined by series or integrals and their analytic continuations. Uniqueness of analytic continuations and applications. Continuous continuation theorem and Schwarz's principle. [3]

- The Gamma Function: Definition of G(z) as an integral. The functional relation. Analytic continuation of G(z), its poles and residues. The reflection formula.
- Fourier and Laplace Transforms:. Integral transforms in general. Fourier's integral theorem. Functions defined on [0, â'ž), the Fourier cosine and sine transforms and their inverses. The complex Fourier transform and its inverse. Extension to the case in which the transform variable is complex and the inverse transform is a contour integral. The Laplace transform and its relationship to the complex Fourier transform. The Bromwich integral inversion formula. Examples of all of these. [4]
- Applications of Integral Transforms to Partial Differential Equations:. A simple linear ODE solved by Laplace transform. Initial value problem for the one-dimensional heat equation for the infinite bar. Same for the semi-infinite bar with appropriate end conditions. The semi-infinite bar with prescribed end temperature. Boundary value problems for Laplaceâ€'s equation in an infinite strip. Same for Helmholz's equation if time permits. [5]

Recommended reading

A standard source for the underlying complex variable theory is

- E.T. Copson, Functions of a Complex Variable, 1995.

For problems solved by integral transforms see

- I.N. Sneddon, The Use of Integral Transforms McGraw Hill, 1972.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 67 hours

Teaching staff

Michael Simon - Unit coordinator