MATH34011

Asymptotic Expansions & Perturbation Methods

Unit code:	MATH34011
Credit Rating:	10
Unit level:	Level 3
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

None

Aims

To demonstrate the power and the beauty of modern asymptotic methods via discussing the main ideas and approaches used in the theory of asymptotic expansions to simplify and to solve different mathematical problems which involve large or small parameters.

Overview

The development of the theory of asymptotic expansions, which serves as a foundation for perturbation methods, is one of the most important achievements in applied mathematics in the twentieth century. Perturbation methods represent a very powerful tool in modern mathematical physics and in particular, in fluid dynamics. This course unit will introduce students to a range of modern asymptotic techniques and illustrate their use in model problems involving ordinary and partial differential equations.

Assessment methods

- Other - 20%
- Written exam - 80%

Assessment Further Information

- Mid-semester coursework: weighting 20%
- End of semester examination: two hours weighting 80%

Learning outcomes

On successful completion of the course unit the students will acquire thorough understanding of fundamental ideas used in the theory of asymptotic expansions and will develop appropriate practical skill in applying asymptotic methods for analysing mathematical and physical problems with small or large parameters.

Syllabus

- Asymptotic expansions. Taylor expansion as a conventional converging power series and as an example of an asymptotic expansion . Asymptotic expansions for definite integrals with the upper or lower limits of integration depending on small or large parameters. Functions defined by real integrals. Laplace's method for definite integrals the integrand being of the form f(t)exp(Î»t), where the parameter Î» is large; Watson's Lemma. Generalisation for functions defined by contour integrals. Steepest descent. Applications.
- Regular asymptotic expansions for functions depending on the coordinate x, scalar or vector, and on a small parameter Îµ. Solution of ordinary and partial differential equations with small parameters.
- Singular perturbations (algebraic example). Notion of the boundary layer. Inner and outer solutions. Overlap region. Matching of the asymptotic expansions. Ordinary differential equations with singular perturbations. Methods to determine location of the boundary layer.
- Method of multiple scales. Quasi-periodic solutions of second order ordinary differential equations developing non-uniformity at large time. Uniformly valid solutions. Amplitude equations. WKB Method.

Recommended reading

- R. Wong, Asymptotic Approximation of Integrals, Academic Press 1989.
- E.J. Hinch, Perturbation Methods, Cambridge 1991.
- O.M. Bender and S.A. Orszag, Advanced Mathematical Methods for Scientists and Engineers.
- M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press 1964.
- J. Kevorkian and J.D. Cole, Perturbation Methods in Applied Mathematics, Springer 1985.

- A.H. Nayfeh, Perturbation Methods, Wiley 1973.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 24 hours
- Tutorials - 12 hours
- Independent study hours - 64 hours

Teaching staff

Jitesh Gajjar - Unit coordinator