MATH47112

Brownian Motion

Unit code:	MATH47112
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 2
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20701 - Probability 2 (Compulsory)

Additional Requirements

MATH47112 pre-requisites

Students are not permitted to take, for credit, MATH47112 in an undergraduate programme and then MATH67112 in a postgraduate programme at the University of Manchester, as the courses are identical.

Aims

The unit aims to provide the basic knowledge necessary to pursue further studies/applications where Brownian motion plays a fundamental role (e.g. Financial Mathematics).

Overview

Brownian motion is the most important stochastic process. It was observed by Brown in 1828 and explained by Einstein in 1905. A more accurate model based on work of Langevin from 1908 was

introduced by Ornstein and Uhlenbeck in 1930. The assumption of stationary independent increments made by Einstein in 1905 has had a profound influence on the development of probability theory in the 20th century. The course unit presents basic facts and ideas of Brownian motion paying particular attention to the issues of dynamics.

Assessment Further Information

End of semester examination: three hours weighting 100%.

Learning outcomes

On successful completion of this course unit students will

- understand the concept of Brownian motion and diffusion processes;
- understand the exact relation of these processes to PDEs with boundary conditions;
- be able to apply these relations to exploit the interplay between probability and analysis.

Future topics requiring this course unit

None.

Syllabus

1. The heat equation (Fourier's law). [1 lecture]
2. The diffusion equation (Fick's law). [1]
3. Einstein's derivation of the diffusion equation (stationary independent increments). [2]
4. The Wiener process (position of a Brownian particle). [6]
5. The Ornstein-Uhlenbeck process (velocity of a Brownian particle). [2]
6. Strong Markov property (starting afresh at stopping times). [2]
7. Diffusion processes (scale function, speed measure, infinitesimal operator). [8]
8. Boundary classification (regular, exit, entrance, natural). [2]
9. The Kolmogorov forward and backward equations. [2]
10. Probabilistic solutions of PDEs (elliptic and parabolic). [6]
11. Optimal stopping, free boundary problems, the American option problem. [2]
12. Optimal stochastic control, the Hamilton-Jacobi-Bellman equation, the optimal consumption-investment problem. [2]

Recommended reading

- Rogers, L. C. G. and Williams, D., Diffusions, Markov Processes and Martingales, Vol. 1 and 2, Cambridge University Press 2000.
- Revuz, D. and Yor, M., Continuous Martingales and Brownian Motion, Springer 1999.
- Karatzas, I. and Shreve, S. E., Brownian Motion and Stochastic Calculus, Springer 1991.
- Karlin, S. and Taylor, H. M., A Second Course in Stochastic Processes, Academic Press 1981.
- Nelson, E., Dynamical Theories of Brownian Motion, Princeton University Press 1967.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 33 hours
- Tutorials - 11 hours
- Independent study hours - 106 hours

Teaching staff

Denis Denisov - Unit coordinator