MATH45061

Continuum Mechanics

Unit code:	MATH45061
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH35001 - Viscous Fluid Flow (Recommended)
- MATH35021 - Elasticity (Recommended)
- MATH20401 - Partial Differential Equations and Vector Calculus A (Compulsory)

Additional Requirements

MATH45061pre-requisites

Students are not permitted to take, for credit, MATH45061 in an undergraduate programme and then MATH65061 in a postgraduate programme at the University of Manchester, as the courses are identical.

Aims

The course unit concerns the formulation and solution of problems in continuum mechanics (solid and fluid mechanics) from a modern unified perspective. The aims are (i) to introduce students to the general analytic machinery of tensor calculus, variational principles and conservation laws in

order to formulate governing equations; and (ii) to be aware of exact, approximate and numerical methods to solve the resulting equations.

Overview

This unit describes the fundamental theory of continuum mechanics in a unified mathematical framework. The unit will cover theories of nonlinear and linear elasticity together with those of compressible and incompressible fluid mechanics.

Assessment methods

- Other - 25%
- Written exam - 75%

Assessment Further Information

- Mid-semester coursework: 25%
- End of semester examination: three hours weighting 75%

Learning outcomes

On successful completion of this course unit students will

- Be able to formulate governing equations for a variety of problems in continuum mechanics.
- Understand the relationship between the general theory and its specialisation to the equations of linear elasticity and incompressible Newtonian fluid mechanics.
- Solve simple problems in continuum mechanics analytically.
- Be aware of certain numerical techniques that can be applied to problems in continuum mechanics.

Syllabus

- Foundations and Fundamentals [3]. Eulerian and Lagrangian coordinates, vectors and tensors, material/convected derivatives, integral theorems
- Kinematics: Deformation and Flow [3]. Measures of strain, stretch, rotation, compatibility, applications of convective derivative, velocity and vorticity.
- Stress [3]. The continuum hypothesis, Cauchy's stress principle, measures of stress, objectivity.
- Fundamental laws, governing equations and thermodynamics [4]. Conservation laws (mass, linear and angular momentum, energy), Cauchy's equations, variational principles, thermodynamics, constitutive modelling.

- Linear elasticity [3]. Hooke's law, Navier-Lame equations, elastostatics, elastodynamics, Airy stress function.
- Incompressible Newtonian Fluids [3]. Viscosity, potential flow, slow flow, high-speed flow, boundary layers
- Nonlinear elasticity [4]. Hyperelasticity and constitutive models, simple exact solutions, principle of virtual work and basic finite elements.
- Complex fluids [4]. Non-Newtonian behaviour, shear thinning/thickening, viscoelasticity.
- Further advanced topics[3]. Interfaces and Fluid-structure interaction.

Recommended reading

- Spencer, A.J.M, "Continuum Mechanics", Dover
- Gonzalez, O. and Stuart, A.M., "A first course in continuum mechanics", CUP
- Irgens, F., "Continuum Mechanics", Springer

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 117 hours

Teaching staff

Andrew Hazel - Unit coordinator