MATH36041

Essential Partial Differential Equations

Unit code:	MATH36041
Credit Rating:	10
Unit level:	Level 3
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20401 - Partial Differential Equations and Vector Calculus A (Compulsory)
- MATH20411 - Partial Differential Equations and Vector Calculus B (Compulsory)

Additional Requirements

MATH36041 pre-requisites

Students must have taken MATH20401 OR MATH20411

Aims

This course builds on MATH20401 (PDEs and vector calculus) to further develop the rigorous study of PDEs using tools from analysis and numerical analysis.

Overview

We study the well posedness of the classical PDEs by semigroup and weak approximation and rigorously develop numerical approximation by the Galerkin and finite difference method. The module is theoretical and has the flavour of a pure module; proofs are given.

Assessment methods

- Other - 20%
- Written exam - 80%

Assessment Further Information

- One test worth 20%
- Two hour end of semester examination; Weighting within unit 80%

Learning outcomes

On completion of this unit successful students will be able to:

- Precisely formulate the notion of solution for a number of important PDEs
- Prove rigorously existence and uniqueness of solution
- Develop the Galerkin method for numerical approximation
- Develop the notion of finite difference approximation

Syllabus

- Introduction. Review of PDEs (elliptic, hyperbolic, parabolic). Finite difference method and convergence (by maximum principle) [4 lecture]
- Elliptic PDEs and weak solutions. Hilbert spaces, inner product, Cauchy-Schwarz, and L2. Definition of weak derivative and weak solution. Examples. Riesz representation theorem and Lax Milgram Lemma. Proof of existence and uniqueness for model diffusion problem. More general models. [6 lectures]
- Galerkin method. Best approximation in the energy norm. Finite element and spectral Galerkin. Rates of convergence. Iteration methods. Comparison with finite difference method. [6 lectures]
- Semigroups of operators. Examples (linear test equation, heat equation on bounded domain, wave equation). Definition by Fourier analysis. Mild solutions. Semilinear equations, existence and uniqueness by contraction mapping. Reaction diffusion equation. Method of lines. Proof of convergence. [6 lectures]

Recommended reading

- Endre Suli and David Mayers, An Introduction to Numerical Analysis, Cambridge University Press, 2003.
- Y. Pinchover and J. Rubinstein, An Introduction to Partial Differential Equations, Cambridge University Press, 2005.
- J. Robinson, Infinite Dimensional Dynamical Systems, Cambridge University Press, 2001.
- K. Morton and Mayers, Numerical Solution of Partial Differential Equations, Cambridge University Press, 2005.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 67 hours

Teaching staff

Sean Holman - Unit coordinator