The University of Manchester

MATH41011

Fourier Anal & Lebesgue Integration

Unit code:	MATH41011
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20101 - Real and Complex Analysis (Compulsory)
- MATH20111 - Real Analysis (Compulsory)
- MATH20122 - Metric Spaces (Compulsory)

Additional Requirements

MATH41011 pre-requisites

Students must have taken MATH20122 and (MATH20101 OR MATH20111)

Students are not permitted to take more than one of MATH31011, MATH41011 or MATH61011 for credit, either in the same or different undergraduate year or in an undergraduate programme and then a postgraduate programme, as the contents of the courses overlap significantly.

Aims

To give an introduction to Lebesgue's theory of measure and integration on the set of real numbers R. To use this to find an appropriate setting in which to understand the convergence of Fourier series.

Overview

It is often convenient to represent functions as Fourier series. However, the convergence of such series is a delicate issue closely related to the theory of integration. A standard approach to integration on the real line, formalised by Riemann, is based on partitioning the domain into smaller intervals. This approach works in many situations but there are simple examples for which it fails. In the early 1900s, H. Lebesgue produced a better theory in which the key idea is to extend the notion of length from intervals to more complicated subsets of R. This started an area of mathematics it its own right, called Measure Theory. Most generally, this is about how one may sensibly assign a size to members of a collection of sets. One application of Lebesgue's ideas is that one can introduce a vector space of functions in which Fourier series appear in a natural way.

This course will appeal to students who have enjoyed MATH20101 or MATH20111 and MATH20122. It will be useful to student taking probability courses in years three and four since the ideas of measure theory have a central role in probability theory.

Assessment methods

- Other - 10%
- Written exam - 90%

Assessment Further Information

- Mid-semester coursework: weighting 10%
- End of semester examination: three hours weighting 90%

Learning outcomes

On successful completion of this course unit students will

- understand how Lebesgue measure on R is defined,
- understand how measures may be used to construct integrals,
- know the basic convergence theorems for the Lebesgue integral,
- understand the relation between Fourier series and the Hilbert space of square integrable functions.

Syllabus

- Fourier series, convergence and Dirichlet's Theorem. [3 lectures]
- Revision of countable and uncountable sets, the Cantor set. [2 lectures]

- Riemann's approach to integration. Lesbesgue measure on R, Borel sets, measurable sets and functions, construction of the Lebesgue integral. [8 lectures]
- Limit theorems for the Lebesgue integral. [3 lectures]
- Square integrable functions and Fourier series, Hilbert spaces. [6 lectures]

The lectures will be enhanced by additional reading on the costruction of Lebesgue measure, existence of non-measurable sets, and general measures. Reading material will be provided.

Recommended reading

- J. Franks, A (Terse) Introduction to Lebesgue Integration, American Mathematical Society, Student Mathematical Library, 2009.
- H. S. Bear, A Primer of Lebesgue Integration, Academic Press, 1995.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 117 hours

Teaching staff

Alex Wilkie - Unit coordinator