The University of Manchester

MATH42122

Galois Theory

Unit code:	MATH42122
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 2
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20212 - Algebraic Structures 2 (Compulsory)
- MATH42001 - Group Theory (Compulsory)
- MATH32001 - Group Theory (Compulsory)

Additional Requirements

Students must have taken MATH20212 and (MATH32001 OR MATH42001).

It is recommended that Year 3 students consult with the lecturer before signing up for this course.

Students are not permitted to take, for credit, MATH42122 in an undergraduate programme and then MATH62122 in a postgraduate programme at the University of Manchester, as the courses are identical.

Aims

To introduce students to a sophisticated mathematical subject where elements of different branches of mathematics are brought together for the purpose of solving an important classical problem.

Overview

Galois theory is one of the most spectacular mathematical theories. It establishes a beautiful connection between the theory of polynomial equations and group theory. In fact, many fundamental notions of group theory originate in the work of Galois. For example, why are some groups called 'soluble'? Because they correspond to the equations which can be solved! (Solving here means there is a formula involving algebraic operations and extracting roots of various degrees that expresses the roots of the polynomial in terms of the coefficients.) Galois theory explains why we can solve quadratic, cubic and quartic equations, but no formulae exist for equations of degree greater than 4. In modern language, Galois theory deals with 'field extensions', and the central topic is the 'Galois correspondence' between extensions and groups. Galois theory is a role model for mathematical theories dealing with 'solubility' of a wide range of problems.

Assessment methods

- Other - 20%
- Written exam - 80%

Assessment Further Information

- Mid-semester coursework: one in-class test, weighting 20%
- End of semester examination: three hours weighting 80%

Learning outcomes

On successful completion of this course unit students will

- have deepened their knowledge about fields;
- have acquired sound understanding of the Galois correspondence between intermediate fields and subgroups of the Galois group;
- be able to compute the Galois correspondence in a number of simple examples;
- appreciate the insolubility of polynomial equations by radicals.

Future topics requiring this course unit

None.

Syllabus

1. Introduction and preliminaries: fields, vector spaces, homogeneous linear systems, polynomials. [4 lectures]
2. Field extensions, algebraic elements, Kronecker's construction. [4]
3. Splitting fields. [1]

4. Group characters, automorphisms and fixed fields. [2]
5. Normal extensions, separable polynomials, formal derivatives. [3]
6. The Fundamental Theorem of Galois Theory, Galois groups of polynomials, examples of the Galois correspondence. [3]
7. Finite fields, roots of unity, Noether's equations. [2]
8. Kummer extensions, [2]
9. Solutions of polynomial equations by radicals and an insolvable quintic. [2]

Recommended reading

- E. Artin, Galois Theory, Dover Publications 1998.
- I Stewart, Galois Theory, 2nd edition, Chapman and Hall.
- J B Fraleigh, A First Course in Abstract Algebra, 5th edition, Addison-Wesley 1967.

Feedback methods

Tutorials will provide an opportunity for students’ work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 117 hours

Teaching staff

Ralph Stohr - Unit coordinator