MANCHESTER 1824
The University of Manchester

MATH43042

Godel's Theorems

Unit code:	MATH43042
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 2
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

None

Additional Requirements

Students are not permitted to take, for credit, MATH43042 in an undergraduate programme and then MATH63042 in a postgraduate programme at the University of Manchester, as the courses are identical.

Aims

To introduce the student to two incompleteness theorems and to their most important corollaries.

Overview

Can a computer be programmed to generate all true statements of mathematics and no false ones? An extraordinary result, proved by Kurt Godel in 1931, shows that even if we restrict ourselves to the most fundamental part of mathematics, namely statements about the natural numbers, the answer to the above question is no. Moreover Godel's method is such that given any "mechanical procedure" for generating true statements about the natural numbers we are actually able to construct a true statement about the natural numbers which will not be generated by that procedure. Later results have shown that such statements can always be chosen to have the simple logical form of an assertion that a particular polynomial equation has no solution over the integers.

Godel's work stands as one of the great landmarks of 20th century thought and has a significance which reaches well beyond mathematics. The course unit will be centred on proofs of Godel's two incompleteness theorems and will examine some of their principal applications.

Assessment Further Information

- Mid-semester coursework: two take home tests weighting 20%
- End of semester examination: two and a half hours weighting 80%

Learning outcomes

On successful completion of this course unit students will

- be familiar with the notion of recursive function;
- have developed a facility in the manipulation and application of these functions, with particular emphasis on applications to logic and decision problems.

Syllabus

1.The completeness theorem for the predicate calculus: a review. First order theories. [2 lectures]

2.Recursive functions and relations. Basic properties. Primitive recursion. Closure under bounded quantification. [4]

3.Gödel's β-function. Coding of finite sequences. Recursively enumerable sets and the arithmetic hierarchy. Church's Thesis. [5]

4.Formal arithmetic. The systems S and PA. Representability. Gödel numbering and the arithmetization of logic. The recursiveness of the proof predicate. Gödel first incompleteness theorem. Tarskis undefinability theorem. [6]

5.Applications of the incompleteness theorem to show the undecidability of the predicate calculus and other axiom systems. More about PA. [8]

6.Kleenes Enumeration Theorem. Hilberts 10th problem. The MRDP theorem and its corollaries. [3]

7.Gödel's second incompleteness theorem. The philosophical impact of Godels work. The limitations of the axiomatic method. [2]

Recommended reading

There is no set textbook covering all the material in the module, although the first two books listed below cover much of it, albeit using different notation. Full lecture notes are provided online on the lecturer's website.

- H.B. Enderton, A Mathematical Introduction to Logic, (especially chapter 3), Academic Press,
- J.R. Shoenfield, Mathematical Logic, (chapters 4 and 6). Addison-Wesley 1967,
- Peter Smith, An Introduction to Godels Theorems, Cambridge University Press, 2007,
- Hao Wang, A Logical Journey, MIT Press, 1997,
- Hao Wang, From Mathematics to Philosophy. RKP 1974,
- G.T. Kneebone, Mathematical Logic and the Foundations of Mathematics. Van Nostrand 1963.

Feedback methods

Tutorials will provide an opportunity for students’ work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 33 hours
- Tutorials - 11 hours
- Independent study hours - 106 hours

Teaching staff

Marcus Tressl - Unit coordinator