MANCHESTER 1824
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MATH42001

Group Theory

Unit code:	MATH42001
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Anti-requisite

- MATH62001 - Group Theory (Compulsory)
- MATH32001 - Group Theory (Compulsory)

Prerequisite

- MATH20212 - Algebraic Structures 2 (Compulsory)

Additional Requirements

MATH42001 pre-requisites

Students are not permitted to take more than one of MATH32001, MATH42001 or MATH62001 for credit, either in the same or different undergraduate year or in an undergraduate programme and then a postgraduate programme, as the contents of the courses overlap significantly.

Aims

This lecture course unit aims to introduce students to some more sophisticated concepts and results of group theory as an essential part of general mathematical culture and as a basis for further study of more advanced mathematics.

Overview

The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outstanding beauty. It takes just three simple axioms to define a group, and it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts and results of Group Theory.

Assessment methods

- Other - 7%
- Written exam - 93%

Assessment Further Information

- Coursework: in-class test weighting 7%
- End of semester examination: three hours, weighting 93%

Learning outcomes

On successful completion of this course unit students will have acquired

- a sound understanding of the classification of finitely generated abelian groups,
- knowledge of some fundamental results and techniques from the theory of finite groups,
- knowledge of group actions on sets, simple groups, Sylow's theorems and various applications of Sylow's theorems.

Syllabus

- Revision of basic notions (subgroups and factor groups, homomorphisms and isomorphisms), generating sets, commutator subgroups. [2 lectures]
- Abelian groups, the Fundamental Theorem on finitely generated abelian groups. [4]
- The Isomorphism Theorems. [3]
- Simple groups, the simplicity of the alternating groups. [3]
- Group actions on sets, orbits, stabilizers, the number of elements in an orbit, Burnside's formula for the number of orbits, conjugation actions, centralizers and normalizers. [5]

- Sylow's Theorems, groups of order pq, pqr. [3]

The lectures will be enhanced by additional reading on related topics:

- commutators and some of their elementary properties;
- proof of Ito's theorem which states that a group which is the product of two abelian groups has trivial second derived subgroup;
- finite abelian groups, including a proof that the rank and torsion coefficients determine the abelian group uniquely;
- proving that A_n for n greater than or equal to 5 is a simple group;
- proving that a simple group of order 60 must be isomorphic to A_5.

Recommended reading

Recommended text:

- John B Fraleigh, A First Course in Abstract Algebra, (5th edition), 1967, Addison-Wesley.

Feedback methods

Tutorials will provide an opportunity for students’ work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 117 hours

Teaching staff

Peter Rowley - Unit coordinator