The University of Manchester

MATH31051

Introduction to Topology

Unit code:	MATH31051
Credit Rating:	10
Unit level:	Level 3
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20122 - Metric Spaces (Recommended)

Additional Requirements

Students are not permitted to take more than one of MATH31051, MATH41051 or MATH610510 for credit, either in the same or different undergraduate year or in an undergraduate programme and then a postgraduate programme, as the contents of the courses overlap significantly.

Aims

This lecture course unit aims to introduce students to the basic concepts of topological spaces and continuous functions, and to illustrate the techniques of algebraic topology by means of the fundamental group.

Overview

This course unit is concerned with the study of topological spaces and their structure-preserving functions (continuous functions). Topological methods underpin a great deal of present day mathematics and theoretical physics. Topological spaces are sets which have sufficient structure so that the notion of continuity may be defined for functions between topological spaces. This structure is not defined in terms of a distance function but in terms of certain subsets known as open subsets which are required to satisfy certain basic properties. Continuous functions may stretch or bend a space and so two spaces are considered to be topologically equivalent if one can be obtained from the other by by stretching and bending: for this reason topology is sometimes called rubber sheet geometry.

The first half of the course unit introduces the basic definitions and standard examples of topological spaces as well as various types of topological spaces with good properties: pathconnected spaces, compact spaces and Hausdorff spaces . The second half introduces the fundamental group and gives some standard applications of the fundamental group of a circle.

Assessment methods

- Other - 15%
- Written exam - 85%

Assessment Further Information

- Mid-semester coursework: weighting 15%
- End of semester examination: two hours weighting 85%

Learning outcomes

On successful completion of this course unit students will be able to

- prove that certain subsets of Euclidean space are homeomorphic (topologically equivalent) by constructing a homeomorphism;
- understand the notions of path-connectedness and path-component and be familiar with some standard applications;
- determine whether a collection of subsets of a set determines a topology;
- use the definitions of the subspace topology, the product topology and the quotient topology, understand the use and proof of their universal properties and be familiar with standard examples such as topological surfaces;
- recognize whether or not a subset of a topological space is compact and be familiar with the basic properties of compact subsets and their proofs;

- recognize whether or not a topological space is Hausdorff and be familiar with the basic properties of Hausdorff spaces and their proofs;
- understand the definition of the fundamental group and its functorial properties;
- calculate the fundamental group of the circle using its univeral covering space and have seen some standard applications.

Syllabus

1. Topological equivalence: the topological equivalence of subsets of Euclidean spaces, path-connected sets and distinguishing subsets of Euclidean spaces using the cut point principle. Standard applications of path-connectedness such as the Pancake Theorem. [3 lectures]
2. Topological spaces: definition of a topology on a set, a topological space and a continuous function between topological spaces; closed subsets of a topological space; a basis for a topology. [2]
3. Topological constructions: subspaces, product spaces, quotient spaces; definitions and basic properties; standard examples including the cylinder, the torus, the Möbius band, the projective plane and the Klein bottle. [5]
4. Compactness: open coverings and subcoverings, definition of a compact subset of a topological space; basic properties of compact subsets; compact subsets in Euclidean spaces (the Heine-Borel Theorem). [2]
5. Hausdorff spaces: definition and basic properties of Hausdorff spaces; a continuous bijection from a compact space to a Hausdorff space is a homeomorphism, quotient spaces of compact Hausdorff spaces. [2]
6. The fundamental group: equivalent paths, the algebra of paths, definition of the fundamental group and dependence on the base point. [3]
7. The fundamental group of the circle: the path lifting theorem for the standard cover of the circle, the degree of a loop in the circle, the fundamental group of the circle, standard applications: the Brouwer Non-Retraction Theorem, the Brouwer Fixed Point Theorem, the Fundamental Theorem of Algebra, the Hairy Ball Theorem. [5]

Recommended reading

The first three of the following books contains most of the material in the course with the third a little more advanced than the first two. The fourth book contains most of material in the first half of the course and relates topological spaces to metric spaces.

- M. A. Armstrong. Basic Topology, Springer 1997.

- C. Kosniowski, A First Course in Algebraic Topology, Cambridge University Press 1980.
- J. R. Munkres, Topology, Prentice-Hall 2000 (second edition).
- W. A. Sutherland, Introduction to Metric and Topological Spaces, Oxford University Press 2009 (second edition).

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 67 hours

Teaching staff

Peter Eccles - Unit coordinator