MATH31002

Linear Analysis

Unit code:	MATH31002
Credit Rating:	10
Unit level:	Level 3
Teaching period(s):	Semester 2
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH20101 - Real and Complex Analysis (Compulsory)
- MATH20111 - Real Analysis (Compulsory)
- MATH20122 - Metric Spaces (Compulsory)

Additional Requirements

MATH31002 pre-requisites

Students must have taken MATH20122 and (MATH20101 OR MATH20111)

Students are not permitted to take more than one of MATH31002, MATH41002 or MATH61002 for credit, either in the same or different undergraduate year or in an undergraduate programme and then a postgraduate programme, as the contents of the courses overlap significantly.

Aims

To give an introduction to Modern Analysis, including elements of functional analysis. The emphasis will be on ideas and results.

Overview

This course unit deals with a coherent and elegant collection of results in analysis. The aim of this course is to provide an introduction to the theory of infinite dimensional linear spaces, which is not only an important tool, but is also a central topic in modern mathematics.

This area has many applications to other areas in Pure and Applied Mathematics such as Dynamical Systems, C* algebras, Quantum Physics, Numerical Analysis, etc.

Assessment Further Information

End of semester examination: two hours weighting 100% (MATH31002), three hours weighting 100% (MATH41002)

Learning outcomes

On successful completion of this module students will be able to

- understand the approximation of continuous functions using the Stone-Weierstrass Theorem;
- understand the concepts of Hilbert and Banach spaces, with l 2 and l p spaces serving as examples;
- understand the definitions of linear functionals and dual spaces, prove the Riesz representation theorem for Hilbert spaces;
- define linear operators, their spectrum (with matrices serving as examples) and their spectral radius, understand the definitions of self-adjoint, isometric and unitary operators on Hilbert spaces and their spectra, be able to apply these ideas to matrices.

Students will have seen proofs of the main results, although an intimate knowledge of the full details is not required.

Future topics requiring this course unit

This course would be useful to students interested in the following topics: MATH41011 Measure and Fractals, MATH41112 Ergodic Theory.

Syllabus

1.Revision of compactness and uniform continuity. [2 lectures]

2.Approximation by polynomials and the Stone-Weierstrass Theorem. [4]

3.Normed vector spaces. Finite and infinite dimensional spaces. Completeness and Banach spaces. [4]

4.Sequence spaces. Continuous function spaces. Separability. Hilbert spaces, orthogonal complements and direct sum decompositions, orthonormal bases. [6]

5.Bounded linear functionals. The Hahn-Banach theorem. Dual spaces. The Riesz representation theorem for Hilbert spaces. [4]

6.Bounded linear operators and their norms. Compact operators. Invertible operators. Spectra. [4]

7.Linear operators on Hilbert spaces. Self-adjoint and unitary operators and their spectra. [4]

Recommended reading

- G. F. Simmons, An Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
- I.J. Maddox, Elements of Functional Analysis, C.U.P. 1989.
- B.P. Rynne and M.A. Youngson, Linear Functional Analysis, Springer S.U.M.S., 2008.

Feedback methods

Tutorials will provide an opportunity for students’ work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 67 hours

Teaching staff

Nikita Sidorov - Unit coordinator