MATH20411

Partial Differential Equations and Vector Calculus B

Unit code:	MATH20411
Credit Rating:	10
Unit level:	Level 2
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH10121 - Calculus and Vectors A (Compulsory)
- MATH10131 - Calculus and Vectors B (Compulsory)

Aims

This course introduces students to analytical and numerical methods for solving partial differential equations (PDEs) and builds on the first year core applied mathematics courses to develop more advanced ideas in differential and integral calculus.

Overview

The main topics to be explored are: Fourier series, partial differential equations, analytical and numerical methods for solving classical PDEs (Laplace's equation and the heat and wave equations) and several topics in vector calculus, including surface and volume integrals. The course covers similar material to MATH20401 but contains a reduced range of topics and with fewer details, where appropriate. The methods employed in the course will prove essential for all of the applied

mathematics and numerical analysis options in the remaining semesters of the Joint Honours BSc and MMath degree programmes.

Assessment methods

- Other - 20%
- Written exam - 80%

Assessment Further Information

- Coursework: an in-class test, weighting within unit 20%
- 2 hour end of semester examination: weighting within unit 80%

Learning outcomes

On completion of this unit successful students will be able to:

- convert Cartesian coordinates into cylindrical and spherical coordinates and sketch surfaces expressed in all of these coordinate systems;
- perform partial differentiation of functions of more than one variable and interpret these derivatives physically;
- recognise classical PDEs describing physical processes such as heat diffusion;
- interpret boundary and initial conditions physically;
- understand the basic concept of orthogonal functions;
- compute Fourier series, Fourier sine series and Fourier cosine series of some piecewise continuous functions;
- classify second-order PDEs as being either elliptic, hyperbolic or parabolic;
- solve, analytically, via the method of separation of variables, the heat and wave equations (in one space variable) and Laplace's equation (in two space variables), on rectangular and circular domains;
- solve, numerically, via finite difference schemes, the heat equation in one space variable;
- solve, numerically, convection-diffusion equations using upwind finite differencing;
- compute elements of surface and volume in different coordinate systems;
- evaluate line, surface and volume integrals over a range of domains, using transformations to other coordinate systems where appropriate;
- understand grad, div and curl operator notation and relate some key identities to physical properties of vector fields;

- understand, and interpret physically, the classical Divergence, Green's and Stokes' theorems.

Syllabus

- Introductory material. [3 lectures]

Cartesian, cylindrical and spherical coordinates. Functions of several variables, surfaces. Partial derivatives, chain rule. Partial differential equations, boundary and initial conditions. Integrals of functions of several variables.

- Fourier series. [4 lectures]

Orthogonality. Fourier series and Fourier coefficients. Periodic, even and odd functions. Fourier's theorem. Fourier sine and cosine series.

- Partial Differential Equations. [2 lectures]

Linearity, homogeneity and order of PDEs. Classification of second-order equations. Introduction to the classical equations: Laplace's, heat and wave equations.

- Analytical Solution of PDEs. [4 lectures]

The method of separation of variables. Solving, exactly, initial-value problems for the heat and wave equations. Solving Laplace's equation in both Cartesian and plane polar co-ordinates.

- Numerical Solution of PDEs. [4 lectures]

Solving, approximately, the reaction-diffusion and convection-diffusion equations (ODEs) via finite difference methods.

Solving, approximately, the heat equation in one space variable (PDE). Explicit and implicit numerical schemes.

- Vector Calculus. [5 lectures]

Surfaces, unit vectors, elements of surface/volume. Line, surface and volume integrals. Scalar and vector fields: differential and integral calculus. Grad, div and curl operators and related identities. Classical theorems: Divergence, Stokes' theorems.

Recommended reading

- Morton, K.W., Mayers, D.F, Numerical solution of partial differential equations, Cambridge University Press, 2005.
- James Stewart, Calculus, Early Transcendentals, Thomson, fifth edition (international student edition), 2003.
- R Haberman, Elementary Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, (Third edition) Prentice-Hall, 1998.

- Schey, H. M. Div, Grad, Curl, and all that : an Informal Text on Vector Calculus, New York : W. W. Norton, various editions.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and to provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 67 hours

Teaching staff

Catherine Powell - Unit coordinator