MANCHESTER 1824
The University of Manchester

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MATH43001

Predicate Logic

Unit code:	MATH43001
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 1
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

None

Additional Requirements

Students are not permitted to take more than one of MATH33001, MATH43001 or MATH63001 for credit, either in the same or different undergraduate year or in an undergraduate programme and then a postgraduate programme, as the contents of the courses overlap significantly.

Aims

- To show how reasoning and the notion of a valid argument can be formalized.
- To provide practical means of demonstrating the validity, or otherwise, of arguments or assertions.
- To provide, via the Completeness Theorems, a broader picture and understanding of the nature of mathematics.
- To instill an understanding of syntax and semantics and the roles they play.

Overview

In our everyday lives we often employ arguments to draw conclusions. In turn we expect others to follow our line of reasoning and thence agree with our conclusions. This is especially true in mathematics where we call such arguments ‘proofs’. By why are such arguments or proofs so convincing, why should we agree with their conclusions? What is it that makes them ‘valid’?
In this course we will attempt to formalize what we mean by these notions within a context/language which is adequate to express almost everything we do in mathematics, and much of everyday communication as well. In so doing we will be led to proving Kurt Gödel’s *Completeness Theorem* (1929) which, by clarifying the relationship between proof and truth, is one of the two most philosophically important results in mathematics.

Assessment methods

- Other - 20%
- Written exam - 80%

Assessment Further Information

- Coursework: One take home test with a weighting of 20%.
- End of semester examination: three hours; weighting 80%

Learning outcomes

On successful completion of the course the students will

- appreciate how patterns of reasoning can be formalized semantically and syntactically;
- understand the relationship between truth and proof;
- in simple cases be able to construct formal proofs;
- in simple cases be able to demonstrate, or contradict, the validity of an argument by semantic means.

Syllabus

The course will cover the topics listed below.

- The language of predicate logic: Alphabet, terms, formulas, sentences, complexity of formulas, synactic manipulation of formulas, subformulas [6 lectures]
- Formal proofs: Inference rules, normal form theorems, consistent sets of formula, Deducation Theorem, deductive closure, theories [4 lectures]
- Structures: First order structures and Tarski's definition of truth, satisfaction, logical consequence, correctness of proofs, basic manipulation of structures [5 lectures]

- Term algebras and construction of structures from consistent sets [2 lectures]
- Completeness theorem: We outline a proof that every valid logical expression is formally derivable, i.e. we give a semantic reformulation of proofs which is fundamental to many parts of logic [2 lectures]
- Applications and consequences of the completeness theorem: The compactness theorem, axiomatizability of classes of first order structures [3 lectures]
- Normal form theorems [reading assignment]
- More on term algebras and full proof of the Completeness theorem [reading assignment]
- Applications to Non-standard structures [reading assignment]

Recommended reading

Self contained course notes will be provided. The following also give well written accounts (though using some different notation and order of presentation):

- Enderton, Herbert B., A Mathematical Introduction to Logic. Second edition. Harcourt/Academic Press, Burlington, MA, 2001.
- Mendelson, E. & Rosen, K.H., Introduction to Mathematical Logic. Fifth edition. CRC Press, 2009.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 117 hours

Teaching staff

Marcus Tressl - Unit coordinator