MATH45132

Stability Theory

Unit code:	MATH45132
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 2
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

Prerequisite

- MATH35001 - Viscous Fluid Flow (Recommended)
- MATH20401 - Partial Differential Equations and Vector Calculus A (Compulsory)

Additional Requirements

MATH45132 pre-requisites

Students are not permitted to take, for credit, MATH45132 in an undergraduate programme and then MATH65132 in a postgraduate programme at the University of Manchester, as the courses are identical.

This course is largely self-contained.

Aims

The aim of this course unit is to introduce students to the basic concepts and techniques of modern stability theory, through case studies in fluid mechanics and transport phenomena.

Overview

Many physical systems can become unstable in the sense that small disturbances superimposed on their basic state can amplify and significantly alter their initial state. In this course we introduce the basic theoretical and physical methodology required to understand and predict instability in a variety of situations with focus on hydrodynamic instabilities and on instabilities in reaction-diffusion systems.

Assessment methods

- Other - 25%
- Written exam - 75%

Assessment Further Information

- Mid-semester coursework: 25%
- End of semester examination: two and a half hour weighting 75%

Learning outcomes

Learning Outcomes

On successful completion of this course unit students will

- derive linearised stability equations for a given basic state;
- perform a normal-mode analysis leading to an eigenvalue problems;
- use the ideas of weakly non-linear stability theory in simple systems;
- appreciate the different physical mechanisms leading to instability.

Syllabus

Assuming general mechanics and fluid mechanics in particular (viscous/inviscid), as well as some aspects of dynamical systems as prerequisites for course.

1. Introduction to stability

Nonlinear dynamics. Linear instability versus nonlinear instability. Outline of the basic procedure involved in a linear stability analysis: dispersion relation, marginal stability curve. Role of weakly nonlinear theory, e.g. normal form for pitchfork bifurcation. Global stability.

2. Linear stability analysis: a case study of Rayleigh-Benard convection

Introduction to physical system, Boussinesq equations, dimensional analysis, Basic state, linear theory, normal modes, marginal stability curve: Analytical approach for idealised boundary conditions.

3. Interfacial instabilities

Nonlinear behaviour of interface. Rayleigh-Taylor, Saffman-Taylor and capillary instabilities.

4. Shear flow instabilities

Inviscid/viscous, Squire's theorem. Rayleigh's equation, Rayleigh's inflexion point criterion, Howard's semi-circle theorem, Orr-Sommerfeld equation. Examples: plane Couette flow, plane Poiseuille flow, pipe flow, Taylor-Couette flow. Problems with normal mode analysis. Pseudo-spectrum and non-normality. Absolute and convective instabilities.

5. Stability in reaction diffusion systems.

Stability of propagating fronts.

6. Bifurcation theory

Local bifurcations, normal forms, structural stability.

7. Nonlinear stability theory

Weakly nonlinear theory, derivation of Stuart-Landau equation, Ginzburg-Landau equation

8. Introduction to pattern formation (if time allows)

Stripes, squares and hexagons, three-wave interactions, role of symmetry, long-wave instabilities of patterns: Eckhaus.

9. Advanced topics (by examples only): parametric instabilities

Faraday instability, parametric pendulum, Mathieu equations.

Recommended reading

- P.G. Drazin, Introduction to hydrodynamic stability. Cambridge University Press (2002)
- F. Charru, Hydrodynamic Instabilities. Cambridge University Press (2011)
- P. Manneville, Instabilities, chaos and turbulence. Imperial College Press (2004)

Feedback methods

Tutorials will provide an opportunity for students’ work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 27 hours

- Tutorials - 6 hours
- Independent study hours - 117 hours

Teaching staff

Joel Daou - Unit coordinator