MANCHESTER
1824
The University of Manchester

MATH45122

Transport Phenomena and Conservation Law

Unit code:	MATH45122
Credit Rating:	15
Unit level:	Level 4
Teaching period(s):	Semester 2
Offered by	School of Mathematics
Available as a free choice unit?:	N

Requisites

None

Aims

The aim of the course is to introduce some ideas associated with transport equations and conservation laws, including linear and nonlinear wave propagation, wave steepening, shock formation, diffusion, dispersion and solitons.

Overview

Transport phenomena and conservation laws are ubiquitous and are a crucial element in all models of physical systems. As material is transported waves may form and the course starts with some examples of linear wave propagation. The effects of non-linearity are very important and lead to wave expansion, wave steepening and shock formation, at which there are discontinuous jumps in the solutions. It is shown how these can be modelled using jump conditions at propagating discontinuities and how diffusion and/or dispersion competes against wave steepening to create diffuse non-linear wave fronts and solitons. The formation of non-linear waves will be demonstrated using simple experiments during the lectures. The mathematical theory underlying these systems will be explained in detail, using many examples ranging from gas dynamics and shallow water flows, to granular, two-phase and traffic flows.

Assessment methods

- Other - 25%
- Written exam - 75%

Assessment Further Information

- Mid-semester coursework: weighting 25%
- End of semester examination: three hours weighting 75%

Learning outcomes

On successful completion of the course unit students will be able to

- understand the differences in the solution properties and physics of physical systems governed by the kinematic wave equation, the K - dV equation, Burger's equation and segregation equation, as well as the avalanche and the shallow water equations;
- solve ut + c(u)ux = 0 for given initial data and be able to identify the formation of shocks;
- understand how breaking waves in two-dimensions can be represented in terms of shock waves;
- solve the gas dynamic and shallow water equations using the method of characteristics for simple flows;
- perform a phase plane analysis for the K - dV and related equations to identify travelling wave solutions, solitary wave solutions.

Future topics requiring this course unit

None.

Syllabus

1.The hyperbolic wave utt = c02Î'2u, ut + c0ux = 0; wave forms; Fourier synthesis; dispersion;

C(k) = dw/dk, group velocity; diffusion, e.g. Burger's linear equation ut + c0ux = vuxx.

2.First order wave equation ut + c(u)ux = 0; characteristics; conservation ideas; conservation forms; granular and traffic flow models. Waves in other physical systems.

3.First order equations in two-dimensions; breaking size segregation waves and their representation in terms of shocks.

4.Shallow water wave theory; the nonlinear equations; wave breaking, dam break problems, via characteristics; normal and oblique shocks in granular flows, linearisation and check against linear theory, and linear irrotational theory.

5.Irrotational water wave theory to obtain the Boussinesq equations; steady solutions of the Boussinesq equations; derivation of the Korteweg-de Vries equation from Boussinesq equations; conservation laws for K - dV; analytical solution of K - dV equation; the soliton.

Recommended reading

- P.G. Drazin and R.S. Johnson, Solitons, An Introduction, CUP 1989.
- G.B. Whitham, Linear and Non-linear Waves, Wiley 1974.
- J. Stoker, Water Waves, Wiley Interscience 1957.
- L. Debnath, Nonlinear Water Waves, Academic Press 1994.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Study hours

- Lectures - 22 hours
- Tutorials - 11 hours
- Independent study hours - 117 hours

Teaching staff

John Gray - Unit coordinator