

Course ID 020753

Propositional Logic

Unit coordinator: Gareth Jones

MATH 20302
Credit rating 10
ECTS credits 5

Semester 2

School of Mathematics
Undergraduate

Level 2

FHEQ level ' Middle part of Bachelors'

Marketing course unit overview

Logic is the study of arguments: what they are and what it means to say that they are sound. As such it is central to Mathematics, Philosophy and, to an increasing extent in recent years, Computer Science.

Most of this course unit will deal with the most basic sort of logical argument (i.e., in everyday parlance, what we mean by 'A follows from B'), namely those which depend for their soundness simply on the commonly agreed interpretations of the logical connectives 'not', 'and', 'or' and 'implies'. That is referred to as propositional logic.

We shall characterise the above notion of 'follows' in two fundamentally different ways, firstly in terms of preservation of truth (semantically), and secondly in terms of the formal rules it obeys (proof theoretically, or syntactically). The highlight of the course unit will be the Completeness Theorem for Propositional Logic, which tells us that these two quite different characterisations are equivalent. This is the younger sibling of the Completeness Theorem for Predicate Logic (which is covered in the course, MATH33001, on Predicate Logic). That is a fundamental result for Mathematics which says that one can set up a notion of formal mathematical proof which is strong enough to capture mathematical truth: if something isn't formally provable then there must exist a counterexample. The propositional logic version of that theorem already allows us to introduce the key concepts and to see how a proof of such a general result can be achieved.

In the final few lectures the student is introduced to some basic ideas about predicate logic. This deals with languages which allow the use of variables together with the quantifiers "for all..." and "there exists..." as well as functions, relations and constants. Such languages are of far greater expressive power than those of propositional logic and using them one can, in principle, formalise any mathematical argument. This part of the course unit serves as a brief introduction to these logics with quantifiers and to concepts which will be used in some level 3/4 units in Logic.

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Aims

The course unit aims to introduce the student to the idea of formalising arguments, both semantically and syntactically, and to the fundamental connection between these approaches.

Learning outcomes

On completion of this unit successful students will be able to:

- formalise arguments in propositional logic both semantically and syntactically and understand how these are connected (via the Completeness Theorem);
- in simple cases be able to determine whether 'A follows from B' is true, using a variety of methods;
- be able to interpret formulas and sentences of predicate logic in mathematical structures

Future topics requiring this course unit

The course unit forms a coherent subject on its own, and provides necessary background knowledge for the third and fourth level Logic course units.

Syllabus

1. Motivation, syntax, propositional variables, connectives, propositional terms. [3 lectures]
2. Valuations, logical consequence, logical equivalence, truth tables, satisfiability, Beth Trees. disjunctive normal form, interpolation, adequate sets of connectives. [6]
3. Rules of proof, formal proofs, the Soundness Theorem. [4]
4. The Completeness and Compactness Theorems. [5]
5. Languages for Predicate Logic, structures and interpretations, with examples. [4]

Assessment methods

Other	20%
Written exam	80%

Two take home tests; Weighting within unit 20%
2 hours end of semester examination; Weighting within unit 80%

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and to provide feedback on their understanding.

Requisites

NONE

Available as free choice? N

Recommended reading

Course unit notes will be provided. It will not be necessary to buy any books, but there a number of good books around which the student might enjoy (although they all tend to use substantially different notation, so that they are definitely not alternatives to the course unit notes), for example:

H.B. Enderton, A Mathematical Introduction to Logic, (second edition) Academic Press 2001, ISBN 0122384520.

E. Mendelson, Introduction to Mathematical Logic, Wadsworth and Brooks 1997, ISBN 0534066240.

I. Chiswell and W. Hodges, Mathematical Logic, Oxford University Press 2007, ISBN 0199215626 (more elementary than the two above).

W. Hodges, Logic, Penguin, 2 revised edition 2001, ISBN 0141003146 (aimed at a more general readership; more about the relations with natural language and philosophy)

Scheduled activity hours

Lectures	22
Tutorials	11

Independent study hours 67 hours