Course ID   022567

Algebraic Topology

MATH 31072

Credit rating 10

ECTS credits 5

Unit coordinator: Hendrik Suess

Semester 2

School of Mathematics

Undergraduate

Level 3

FHEQ level ' Last part of a Bachelors'

Marketing course unit overview

The basic method of Algebraic Topology is to associate an algebraic object to each topological space so that homeomorphic spaces have isomorphic algebraic objects. These algebraic objects are then topological invariants and so can be used to distinguish topological spaces. The fundamental group introduced in MATH31051 is such an algebraic object. This course unit introduces the homology groups.

In order to provide motivation for homology theory, the first part of the course unit discusses the classification theorem of closed surfaces using a combinatorial method of proof via simplicial surfaces. Roughly speaking the theorem states that every closed surface can be built up out of the sphere, the torus and the projective plane. Non-homeomorphic surfaces are distinguished using two key topological invariants: the Euler characteristic and orientability.

The second part then generalizes the idea of a simplicial surface to the idea of a simplicial complex and then generalizes this further to a Î'-set (in order to simplify calculations). This enables us to introduce a wide class of topological spaces called polyhedra which can be represented by a Î'-set. This representation can then be used to define the homology groups of the polyhedron. The Euler characteristic and orientability type of a surface are determined by the homology groups. These powerful invariants have many attractive applications.

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Aims

This lecture course has for its aim the further development of the concepts introduced in the course Introduction to Topology, emphasizing topics coming from Algebraic Topology. The course will introduce students to the basic concepts of homotopy and homology theory and explain the need for different algebraic invariants of topological spaces focussing on the classification theorem for closed surfaces.

Learning outcomes

On successful completion of this course unit students will be able to:

- recognize when a collection of triangles in a Euclidean space forms a simplicial surface;
- recognize the underlying space of a simplicial surface by reducing a representing symbol to standard form;
- recognize the underlying space of a simplicial surface by calculating its Euler characteristic and determining whether it is orientable;
- understand the concept of homotopy and know its basic properties;
- recognize whether or not two topological spaces are homotopic in simple cases;
- understand the notions of simplicial complex and delta-set;
- calculate the homology groups of naturally occurring topological spaces;
- understand why homology groups are topological invariants;
- understand the relationship between homology groups and the Euler characterisic and orientability of a simplicial surface;
- use homology groups to say something about the homotopy type of a topological space.

Syllabus

- **Topological surfaces:** definition and basic examples; the connected sum of two surfaces; the classification theorem for compact surfaces; handles and cross-caps. [3]
- **Simplicial surfaces**: definition, triangulation of a topological surface and the statement of the triangulation theorem for compact surfaces; representing the underlying space of a simplicial surface by a symbol; equivalent symbols and the statement and proof of the classification theorem for surface symbols; geometrical interpretation of the classification theorem. [3]
- **Topological invariants of surfaces**: definition of the Euler characteristic and orientability of a simplicial surface; statement of topological invariance; using these invariants to recognize the underlying space of a simplicial surface. [2]
- **Homotopy and homotopy equivalence**: definition and simple examples, deformation retractions. [2]
- Simplicial complexes and delta-sets: definitions and examples; the underlying space, polyhedra. [3]
- **Simplicial homology groups**:definitions and examples. [3]

- **Simplical approximation and topological invariance**: statement of the Simplicial Approximation Theorem and how this may be used to prove the topological invariance of homology groups. [3]
- **Applications:** Brouwer Fixed Point Theorem, Borsuk-Ulam Theorem, Lefschetz Fixed Point Theorem. [2]

Assessment methods

Other	15%
Written exam	85%

Mid-semester coursework: weighting 15%,End of semester examination: two hours weighting 85%.

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Requisites

MATH20212	Algebraic Structures 2	Pre-Requisite	Recommended
MATH31051	Topology	Pre-Requisite	Compulsory

Students are not permitted to take more than one of MATH31072 or MATH41072 for credit in the same or different undergraduate year. Students are not permitted to take MATH41072 and MATH61072 for credit in an undergraduate programme and then a postgraduate programme.

Available as free choice? N

Recommended reading

- M.A. Armstrong, *Basic Topology*, Springer 1997 (classification of surfaces, simplicial complexes, homology)
- A. Hatcher, *Algebraic Topology*. (free download) (simplicial complexes, delta-complexes, higher homotopy groups)
- W.S. Massey, *Algebraic Topology*: an Introduction, Springer 1990 (classification of surfaces)

Scheduled activity hours

Lectures	22
Tutorials	11

Independent study hours 67 hours

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