Course ID 009270

Hyperbolic Geometry

MATH 32051

Credit rating 10

ECTS credits 5

Unit coordinator: Charles Walkden

Semester 1

School of Mathematics

Undergraduate

Level 3

FHEQ level ' Last part of a Bachelors'

Marketing course unit overview

Consider the Euclidean plane *R*. If we take a straight line *L* and a point *p* not on that line, then there is a unique straight line through *p* that never intersects *L* (draw a picture!). This is Euclid's parallel postulate. Euclid introduced several axioms for what is now called Euclidean geometry (that is, geometry in *R* or more generally in*R* and a great deal of effort was employed in attempting to prove that these axioms implied the parallel postulate. However, in early 19th century, the hyperbolic plane was introduced as a setting in which Euclid's axioms hold but the parallel postulate fails: there may be infinitely many "straight" lines through a point that do not intersect a given "straight" line.

Today, hyperbolic geometry is a rich and active area of mathematics with many beautiful theorems (and can be used to generate very attractive pictures)

This course provides an introduction to hyperbolic geometry. We start by discussing what is meant by "distance" and what is "straight" about a straight line in the Euclidean plane *R*. We then give an introduction to the hyperbolic plane. Topics include: distance and area in the hyperbolic plane, distance-preserving maps, hyperbolic trigonometry and hyperbolic polygons.

The collection of all distance-preserving maps forms a group. The second part of the course studies a particular class of such groups, namely Fuchsian groups. By using a very beautiful theorem called Poincaré's Theorem, we will describe the connections between such groups and tessellations (tilings) of the hyperbolic plane. The emphasis here will be on how to calculate with and apply Poincaré's Theorem, rather than on rigorous proofs.

One aim of the course is to show how results and techniques from different areas of mathematics, notably geometry, algebra and analysis, can be used coherently in the study of a single topic.

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Aims
To provide an introduction to the hyperbolic plane and hyperbolic geometry. To study how discrete groups of isometries act on the hyperbolic plane.

Learning outcomes
On successfully completing the course students will be able to:
- understand the geometry of the hyperbolic plane and be able to prove results and identities in hyperbolic geometry and trigonometry,
- understand other models of the hyperbolic plane, such as the Poincaré disc model,
- understand how the group of Möbius transformations acts on the hyperbolic plane by isometries,
- understand how discrete groups of isometries act on the hyperbolic plane and their connections with fundamental domains and Dirichlet regions.

Syllabus
- Introduction, background and motivation.
- The upper half-plane model, hyperbolic distance and area, geodesics. The group of Möbius transformations as isometries.
- The Poincaré disc model. Möbius transformations of the Poincaré disc.
- Hyperbolic triangles, hyperbolic trigonometry, hyperbolic polygons
- Classifying different types of isometries.
- Introduction to discrete groups of isometries.
- Fundamental domains and Dirichlet regions.
- Poincaré's theorem and groups generated by side-pairing transformations.

Assessment methods
Coursework: There is no coursework for this course. However, there will be an 'informal quiz' in week 7/8 based around past exam questions that you can submit for marking and for feedback. Two hour end of semester examination; Weighting within unit 100%

Feedback methods
Tutorials will provide an opportunity for students' work to be discussed and provide feedback

on their understanding.

Requisites
Students are not permitted to take more than one of MATH32051 or MATH42051 for credit in the same or different undergraduate year. Students are not permitted to take MATH42051 and MATH62051 for credit in an undergraduate programme and then a postgraduate programme.

Available as free choice? N

Recommended reading
- J. Anderson, Hyperbolic Geometry, Springer, 1999.
- S. Katok, Fuchsian Groups, Chicago, 1992
- A. Beardon, The Geometry of Discrete Groups, Springer, 1983

The book by Anderson is the most suitable for the course.

Scheduled activity hours

Lectures	22
Tutorials	11

Independent study hours 67 hours

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