Course ID   020997

Differentiable Manifolds

MATH 41061

Credit rating 15

ECTS credits 7.5

Unit coordinator: Theodore Voronov

Semester 1

School of Mathematics

Undergraduate

Level 4

FHEQ level ' Masters/Integrated Masters P4'

Marketing course unit overview

Differentiable manifolds are among the most fundamental notions of modern mathematics. Roughly, they are geometrical objects that can be endowed with coordinates; using these coordinates one can apply differential and integral calculus, but the results are coordinate-independent.

Examples of manifolds start with open domains in Euclidean space Rn , and include "multidimensional surfaces" such as the n-sphere Sn and n-torus Tn , the projective spaces RPn and CPn , and their generalizations, matrix groups such as the rotation group SO(n), etc. Differentiable manifolds naturally appear in various applications, e.g., as configuration spaces in mechanics. They are arguably the most general objects on which calculus can be developed. On the other hand, differentiable manifolds provide for calculus a powerful invariant geometric language, which is used in almost all areas of mathematics and its applications.

In this course we give an introduction to the theory of manifolds, including their definition and examples; vector fields and differential forms; integration on manifolds and de Rham cohomology.

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Aims
The unit aims to introduce the basic ideas of differentiable manifolds.

Learning outcomes
On completion of this unit successful students will be able to:
- deal with various examples of differentiable manifolds and smooth maps;
- have familiarity with tangent vectors, tensors and differential forms;
- work practically with vector fields and differential forms;
- appreciate the basic ideas of de Rham cohomology and its examples;
- apply the ideas of differentiable manifolds to other areas.

Future topics requiring this course unit
Differentiable manifolds are used in almost all areas of mathematics and its applications, including physics and engineering.

Syllabus
1.Manifolds and smooth maps. Coordinates on familiar spaces. Charts and atlases. Definitions of manifolds and smooth maps. Products. Specifying manifolds by equations. More examples of manifolds.

2.Tangent vectors. Velocity of a curve. Tangent vectors. Tangent bundle. Differential of a map.

3.Topology of a manifold. Topology induced by manifold structure. Identification of tangent vectors with derivations. Bump functions and partitions of unity. Embedding manifolds in RN.

4.Tensor algebra. Dual space, covectors and tensors. Einstein notation. Behaviour under maps. Tensors at a point. Example: differential of a function as covector.

5.Vector fields. Tensor and vector fields. Examples. Vector fields as derivations. Flow of a vector field. Commutator.

6.Differential forms. Antisymmetric tensors. Exterior multiplication. Forms at a point. Bases and dimensions. Exterior differential: definition and properties.

7.Integration. Orientation. Integral over a compact oriented manifold. Independence of atlas and partition of unity. Integration over singular manifolds and chains. Stokes theorem.

8.De Rham cohomology. Definition of cohomology and examples of nonzero classes. PoincarÃ© Lemma. Examples of calculation.

Assessment methods

Other	20%
Written exam	80%

Coursework; Weighting within unit 20%3 hours end of semester examination; Weighting within unit 80%

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Requisites

MATH20222	Introduction to Geometry	Pre-Requisite	Recommended
MATH20132	Calculus of Several Variables	Pre-Requisite	Compulsory
MATH41051	Introduction to Topology	Pre-Requisite	Recommended

Students are not permitted to take more than one of MATH31061 or MATH41061 for credit in the same or different undergraduate year. Students are not permitted to take MATH41061 and MATH61061 for credit in an undergraduate programme and then a postgraduate programme.

Available as free choice? N

Recommended reading

No particular textbook is followed. Students are advised to keep their own lecture notes. There are many good sources available treating various aspects of differentiable manifolds on various levels and from different viewpoints. Below is a list of texts that may be useful. More can be found by searching library shelves.

- R. Abraham, J. E. Marsden, T. Ratiu. Manifolds, tensor analysis, and applications.
- B.A. Dubrovin, A.T. Fomenko, S.P. Novikov. Modern geometry, methods and applications.
- A. S. Mishchenko, A. T. Fomenko. A course of differential geometry and topology
- S. Morita. Geometry of differential forms.
- Michael Spivak. Calculus on manifolds.
- Frank W. Warner. Foundations of differentiable manifolds and Lie groups

Scheduled activity hours

Lectures	33
Tutorials	11

Independent study hours 106 hours