Course ID 037532

Symmetry in Nature

MATH 45082

Credit rating 15

ECTS credits 7.5

Unit coordinator: James Montaldi

Semester 2

School of Mathematics

Undergraduate

Level 4

FHEQ level ' Masters/Integrated Masters P4'

Marketing course unit overview

Symmetry arises frequently, both in Nature and in Mathematical models of Nature, and the appreciation of symmetry is deeply ingrained in our consciousness and in our sense of beauty. But symmetry also arises in Mathematics in ways that do not arise from Nature, such as in Galois' analysis of polynomial equations arising from symmetries in their roots. We will start the course by understanding why Group Theory is the natural mathematical language for symmetry, and study some symmetries arising in geometry, such as the symmetry of a cube, consisting of certain rotations and reflections in space which together form a group. We will discuss the classification of symmetries of repeated patterns, like those in the famous Alhambra Mosque in Grenada, giving rise to the so-called Wallpaper Groups. The second half of the course will look more closely at models arising in Nature, in particular through differential equations. Through partial differential equations, the symmetry language allows us to discuss such questions as why is a zebra striped, or a leopard is spotted: both arise through a phenomenon called spontaneous symmetry breaking.

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Aims

To develop an understanding of symmetry as it arises in nature, and to develop the mathematical

techniques for its study through the action of groups.

Learning outcomes
On successfully completing the course, the student will be able to:

• understand and analyze symmetry from a mathematical perspective

• apply the orbit-stabilizer theorem

• analyze the influence of symmetry on symmetric systems

• explain and predict spontaneous symmetry breaking phenomena

Syllabus
1. What is symmetry? Examples. Groups of transformations. Orbits and stabilizers.

2. Symmetry in geometry: Example classification

of triangles

3. Classification: of symmetry groups in 2 and 3 dimensions

4. Symmetry of lattices: (frieze patterns, wallpaper groups and crystals)

5. Symmetry and ODEs: symmetric and nonsymmetric

solutions; spontaneous symmetry

breaking

6. Spatiotemporal

symmetry: (eg, animal gaits)

7. Symmetry and PDEs: pattern formation and more spatiotemporal

symmetry

Assessment methods

Other	12%
Written exam	55%
Project output (not diss/n)	33%

Coursework (worth 12%)End of semester examination (worth 55%)Written project plus short oral presentation (worth 33%)

Feedback methods
Tutorials will provide an opportunity for students' work to be discussed and provide feedback

on their understanding.

Requisites

MATH20201	Algebraic Structures 1	Pre-Requisite	Compulsory

Students are not permitted to take more than one of MATH35082 or MATH45082 for credit in the same or different undergraduate year.

Available as free choice? N

Recommended reading

General:

I.N. Stewart, Symmetry, a very short introduction, Oxford (2013)

H. Weyl Symmetry, Princeton Science Library (1952)

Advanced:

M. Golubitsky & I. Stewart, The Symmetry Perspective, Birkhauser Verlag (2002)

R. Hoyle, Pattern Formation, CUP (2006)

Scheduled activity hours

Assessment practical exam	50
Lectures	22
Tutorials	11

Independent study hours 67 hours

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