Course ID 029294

Applied Dynamical Systems

MATH 64041

Credit rating 15

ECTS credits 7.5

Unit coordinator: Paul Glendinning

Semester 1

School of Mathematics

Postgraduate Taught

Level 6

FHEQ level ' Masters/Integrated Masters P4'

Marketing course unit overview

Dynamical systems theory is the mathematical theory of time-varying systems; it is used in the modelling of a wide range of physical, biological, engineering, economic and other phenomena. This module presents a broad introduction to the area, with emphasis on those aspects important in the modelling and simulation of systems. General dynamical systems are described, along with the most basic sorts of behaviour that they can show. The dynamical systems most commonly encountered in applications are formed from sets of differential equations, and these are described, including some practical aspects of their simulation. The most regular kinds of behaviour---equilibrium and periodic---are the most easy to analyze theoretically; linearization about such trajectories are discussed (for periodic behaviour this is done using the PoincarÃ© map.)

Much more complex behaviours, including chaos, may be found; these are described by means of their attractors. The linearization approach can be extended to these, and leads to the concept of Lyapunov exponents.

In applications it is often important to know how the observed behaviour changes with changes in the system parameters; such changes can often be sudden, but frequently conform to one of a relatively small number of scenarios: the study of these forms the subject of bifurcation theory. The simplest bifurcations are discussed.

Course unit overview

Dynamical systems theory is the mathematical theory of time-varying systems; it is used in the modelling of a wide range of physical, biological, engineering, economic and other phenomena. This module presents a broad introduction to the area, with emphasis on those aspects important in the modelling and simulation of systems. General dynamical systems are described, along with the most basic sorts of behaviour that they can show. The dynamical systems most commonly encountered in applications are formed from sets of differential equations, and these are described, including some practical aspects of their simulation. The most regular kinds of behaviour---equilibrium and periodic---are the most easy to analyze theoretically; linearization about such trajectories are discussed (for periodic behaviour this is done using the PoincarÃ© map.)

Much more complex behaviours, including chaos, may be found; these are described by means of their attractors. The linearization approach can be extended to these, and leads to the concept of Lyapunov exponents.

In applications it is often important to know how the observed behaviour changes with changes in the system parameters; such changes can often be sudden, but frequently conform to one of a relatively small number of scenarios: the study of these forms the subject of bifurcation theory. The simplest bifurcations are discussed.

Aims

To develop a basic understanding dynamical systems theory, particularly those aspects important in applications. To describe and illustrate how the basic behaviours found in dynamical systems may be recognized and analyzed.

Learning outcomes

- On successful completion of this course unit students will
- understand the general concept of a dynamical system, and the significance of dynamical systems for modelling real world phenomena;
- be able to analyze simple dynamical systems to find and classify regular behaviour;
- appreciate some of the more complex behaviours (including chaotic), and understand some of the features of the attractors characterizing such behaviour;
- be familiar with some of the simpler bifurcation scenarios, and how they can be analyzed.

Syllabus

1. **Basics.** Basic concepts of dynamical systems: states, state spaces, dynamics. Discrete and continuous time systems. [1 lecture]

   Some motivating examples: (discrete): simple population models, numerical algorithms; (continuous): chemical and population kinetics, mechanical systems, electronic and biological oscillators. [1]
2. **Basic features of dynamical systems.** Trajectories, fixed points, periodic orbits, attractors and basins. Autonomous and non-autonomous systems. Phase portraits in the plane and higher dimensions; examples of phase portraits of 2-d and 3-d systems. [2]
3. **Ordinary differential equations.** Systems of first order ordinary differential equations; initial value problems, existence and uniqueness of solutions. Flows. [2]
4. **Equilibria and linearization.** Fixed and equilibrium points and their linearization; classification and the Hartman-Grobman theorem; invariant manifolds; examples in 2-d and 3-d. Computing equilibrium points. Stability and Liapounov functions. [5]
5. **Periodic orbits and linearization.** Poincaré sections and the Poincaré map. Linearization and characteristic multipliers of periodic orbits, and stability; examples. Computing periodic orbits. [3]
6. **Attractors and long-term behaviour.** ω-limit sets and long term behaviour. Chaotic attractors; illustrative examples. Lyapunov exponents and their computation. One-dimensional maps and simple routes to chaos (unimodal maps and Lorenz maps). Crises, chaotic transients [8]
7. **Bifurcations of flows.** Local bifurcations and centre manifolds, global bifurcations; examples. Computing bifurcation diagrams by continuation.[8]

Assessment methods

Other	25%
Written exam	75%

Mid-semester coursework: 25% End of semester examination: three hours weighting 75%

Feedback methods

Tutorials will provide a place for student worked examples to be marked and discussed providing feedback on performance and understanding. Feedback is also provided via return of marked coursework.

Requisites

MATH64041	Applied Dynamical Systems	Anti-requisite	Compulsory

Available as free choice? N

Recommended reading

- Stephen~H. Strogatz, Nonlinear Dynamics and Chaos, Perseus Books, Cambridge, MA, USA, 1994.
- Kathleen T. Alligood, Tim D. Sauer and James A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer-Verlag, New York, NY, USA, 1996.
- Thomas S. Parker and Leon O. Chua, Practical Numerical Algorithms for Chaotic Systems, Springer-Verlag, New York, NY, USA, 1989.
- Stephen Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer-Verlag, New York, NY, USA, second edition, 2003.
- Y.A. Kuznetsov Elements of Applied Bifurcation Theory, Springer Applied Math. Sci. 112, 1995.

Scheduled activity hours

Lectures	22
Tutorials	11

Independent study hours 117 hours