Course ID 027281

Linear Models with Nonparametric Regression

MATH 68011

Credit rating 15
ECTS credits 7.5

Unit coordinator: Jianxin Pan
Unit coordinator: Christiana Charalambous

Semester 1

School of Mathematics
Postgraduate Taught

Level 6

FHEQ level ' Masters/Integrated Masters P4'

Marketing course unit overview
In many areas of science, technology, social science and medicine one often wishes to explore the relationship between one observable random response and a number of 'factors' which may influence simultaneously the response. The techniques developed to study such relationships fall in three broad categories:

- Regression Analysis where the influence of the factors is quantitative;
- Analysis of Variance where each factor's influence is qualitative; and
- Analysis of Covariance where both qualitative and quantitative factors are present.

However, these three valuable techniques can be studied together as special cases of a unified theory of Linear Models. The course starts with a study of estimation and hypothesis testing in the general linear problem. Once the principles and techniques are established practical applications in the three types of analysis are examined in greater detail. Nonparametric regression provides a very flexible approach to exploring the relationship between a response and an associated covariate but without having to specify a parametric model. The different techniques available are essentially based on forms of local averaging controlled by the value of a smoothing parameter. In this part of the module we will study a few different techniques, along with their statistical properties. We will also look briefly at how such estimators can be used in more inferential procedures.

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Aims

- To familiarise students with the methodology and applications of standard techniques of Linear Models such as regression analysis, and analysis of variance and covariance, etc.
- To introduce students to advanced statistical methods in Nonparametric Regression.
- To explore some of the wide range of real-life situations occurring in the fields of biology, engineering, industrial experimentation, medical and social sciences that can be investigated using Linear Models and Nonparametric Regression.

Learning outcomes

On successful completion of this course unit students will be able to

- Fit linear models and comment on the adequacy of the fit;
- identify and apply appropriate transformations of either the response or of the covariates;
- check for and identify colinearities, and understand the implications of their presence;
- produce confidence intervals for linear combinations and ratios of linear combinations of the model parameters;
- formulate hypotheses in terms of the model parameters and construct test procedures for testing such hypotheses.
- Be familiar with advanced approaches to estimating a nonparametric regression model;
- understand the role of the smoothing parameter in constructing a nonparametric regression curve;
- know how to derive the asymptotic properties of the chosen estimator;
- be able to use such estimators to check certain parametric assumptions.
- Use the statistical software R to analyze real data using both parametric and nonparametric techniques.

Syllabus

Linear Models

- General Linear Models: Least squares estimators (l.s.e) and their properties. Residuals and residual sum of squares. Leverage. Distribution of l.s.e and of the residual sum of squares. [5]
- The general linear hypothesis. Extra sum of squares, sequential sum of squares, partial sum of squares. The test statistic of the general linear hypothesis and its distribution. Confidence intervals and prediction intervals. [5]
- Linear regression: Simple regression, multiple regression, dummy variables and analysis of covariance. [6]
- Analysis of Variance. One and two way analysis of variance. Use of comparisons. Interactions. [6]

Nonparametric Regression

- Least squares regression, local averaging. [2]
- Local polynomial kernel regression. [3]

- Choosing the value of the smoothing parameter. [1]
- Variability bands, checking the validity of a parametric regression model. [3]
- Introduction to spline regression. [2]

Assessment methods

Other	20%
Written exam	80%

Coursework: weighting 20%End of semester examination: three hours, weighting 80%

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Requisites

MATH20701	Probability 2	Pre-Requisite	Compulsory
MATH20802	Statistical Methods	Pre-Requisite	Compulsory

Students are not permitted to take more than one of MATH38141, MATH48011 or MATH68011 for credit, either in the same or different undergraduate year or in an undergraduate programme and then a postgraduate programme, as the contents of the courses overlap significantly.

Available as free choice? N

Recommended reading

- Weisberg, S., Applied Linear Regression J. Wiley 2005
- Montgomery, D. C. and Peck, E. A., Introduction to Linear Regression Analysis, J. Wiley 2001.
- Rawlings, J. O., Applied Regression Analysis: A Research Tool, Wadsworth and Brooks/Cole 1998.
- Bowman, A. W. and Azzalini, A. Applied Smoothing Techniques for Data Analysis. Oxford University Press (1998)
- Wand, M.P. and Jones, M.C. Kernel Smoothing. Chapman and Hall (1995)
- Eubank, R.L. Spline Smoothing and Nonparametric Regression. Marcel Dekkar
- Hardle, W. Applied Nonparametric Regression. Cambridge University Press (1991)

Scheduled activity hours

Lectures	33
Tutorials	11

Independent study hours 106 hours